Bootstrapping Thermal CFTs

Julien Barrat

23.07.2025

IGST

(Mostly) based on

The analytic bootstrap at finite temperature,

JB, D. N. Bozkurt, E. Marchetto, A. Miscioscia, E. Pomoni,

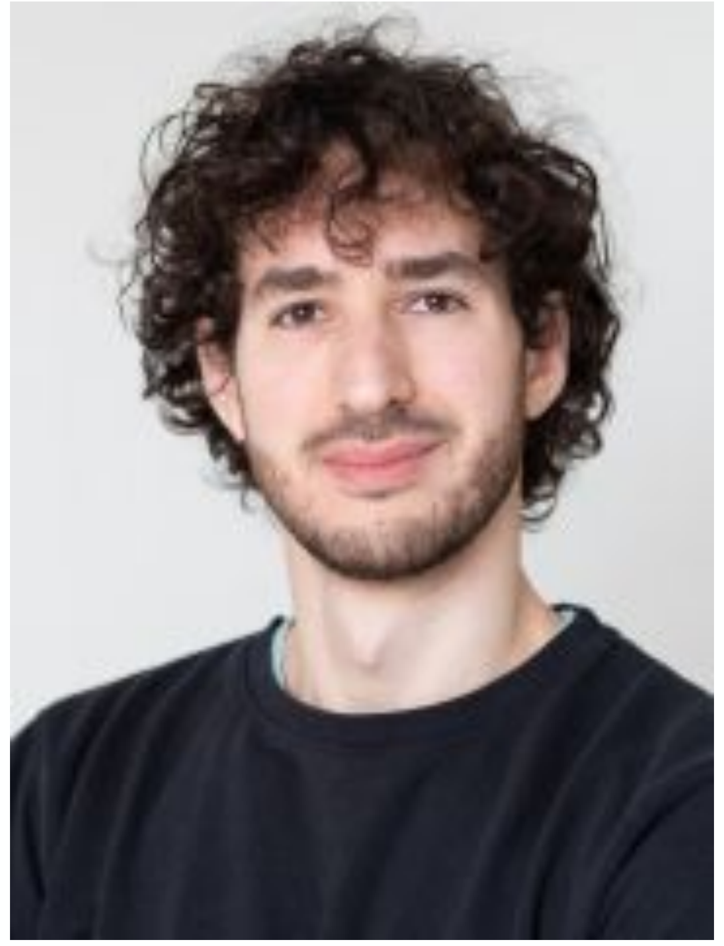
2506.06422.

Collaborators

Elli Pomoni

Deniz Bozkurt

Alessio Miscioscia



Enrico Marchetto

Outline

1. CFT at finite temperature

2. Bootstrapping thermal CFTs

3. Applications

4. Outlook

1. CFT at finite temperature

1. CFT at finite temperature

MOTIVATIONS [Caron-Huot, David, Dodelson, Fitzpatrick, Giombi, Iliesiu, Karlsson, Komargodski, Liu, Maldacena, Pal, Parnachev, Russo, Sachdev, Schomerus, Simmons-Duffin, Skenderis, Son, Starinets, Withers, Witten, Zhiboedov, …]

1. CFT at finite temperature

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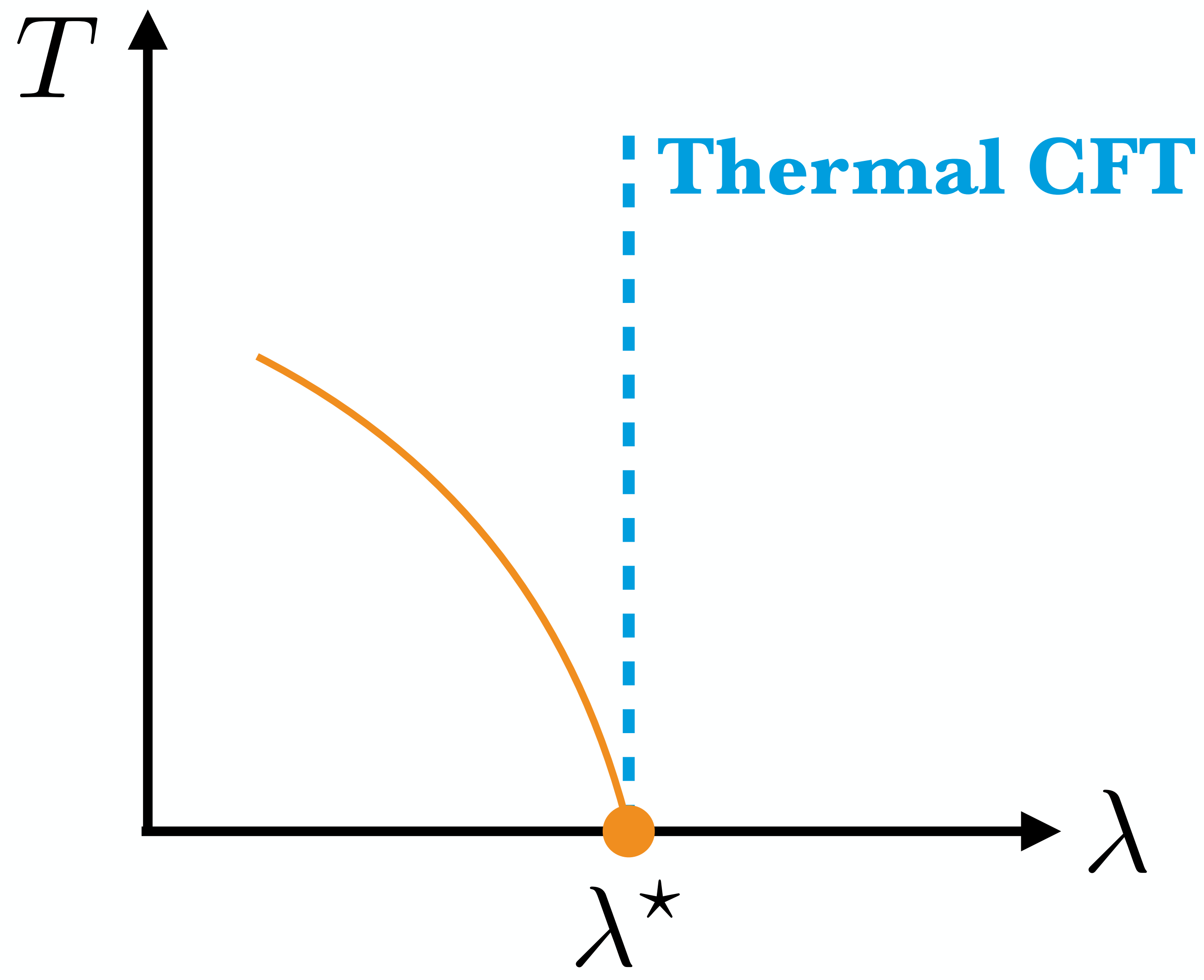
Temperature is unavoidable
in experiments



1. CFT at finite temperature

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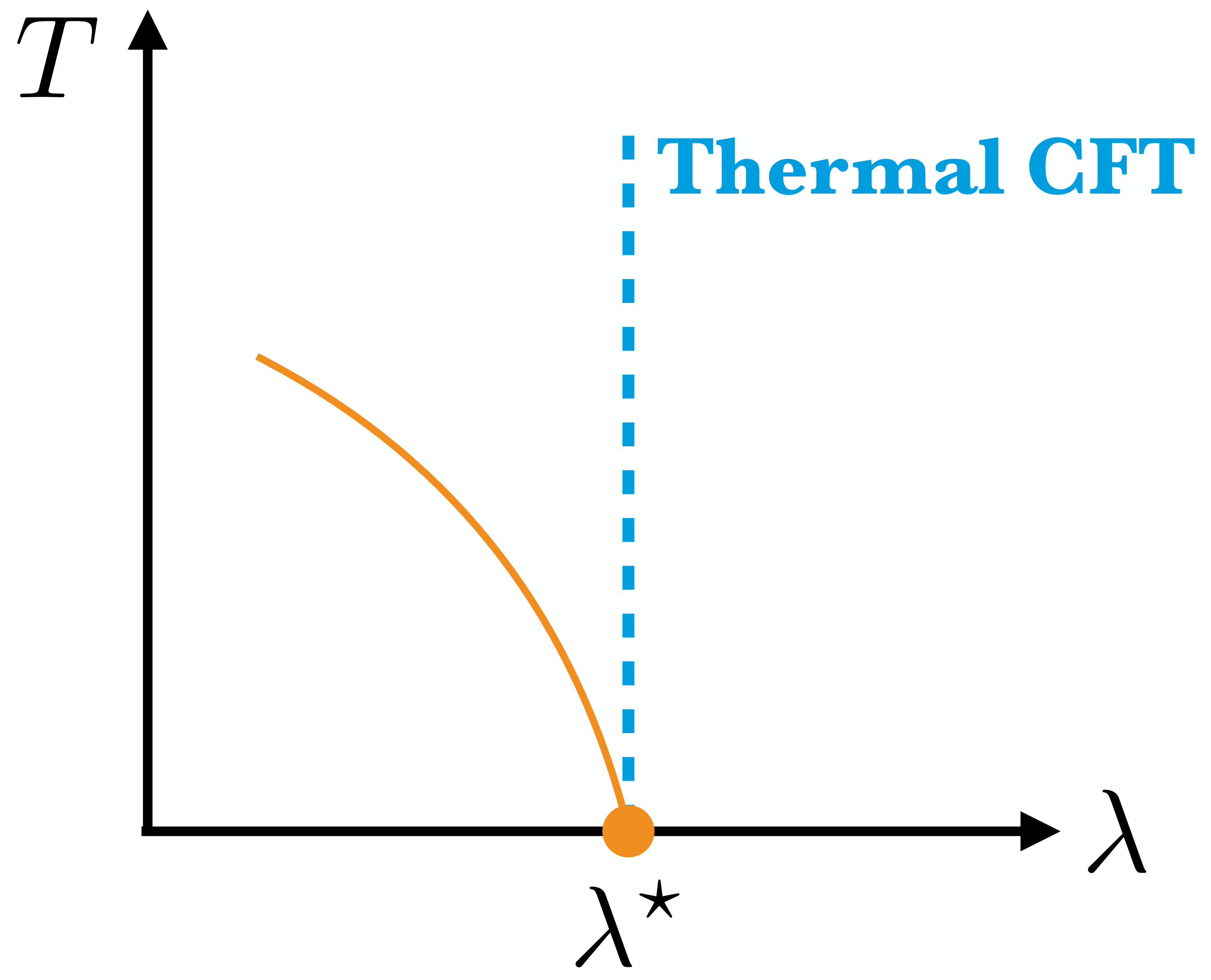
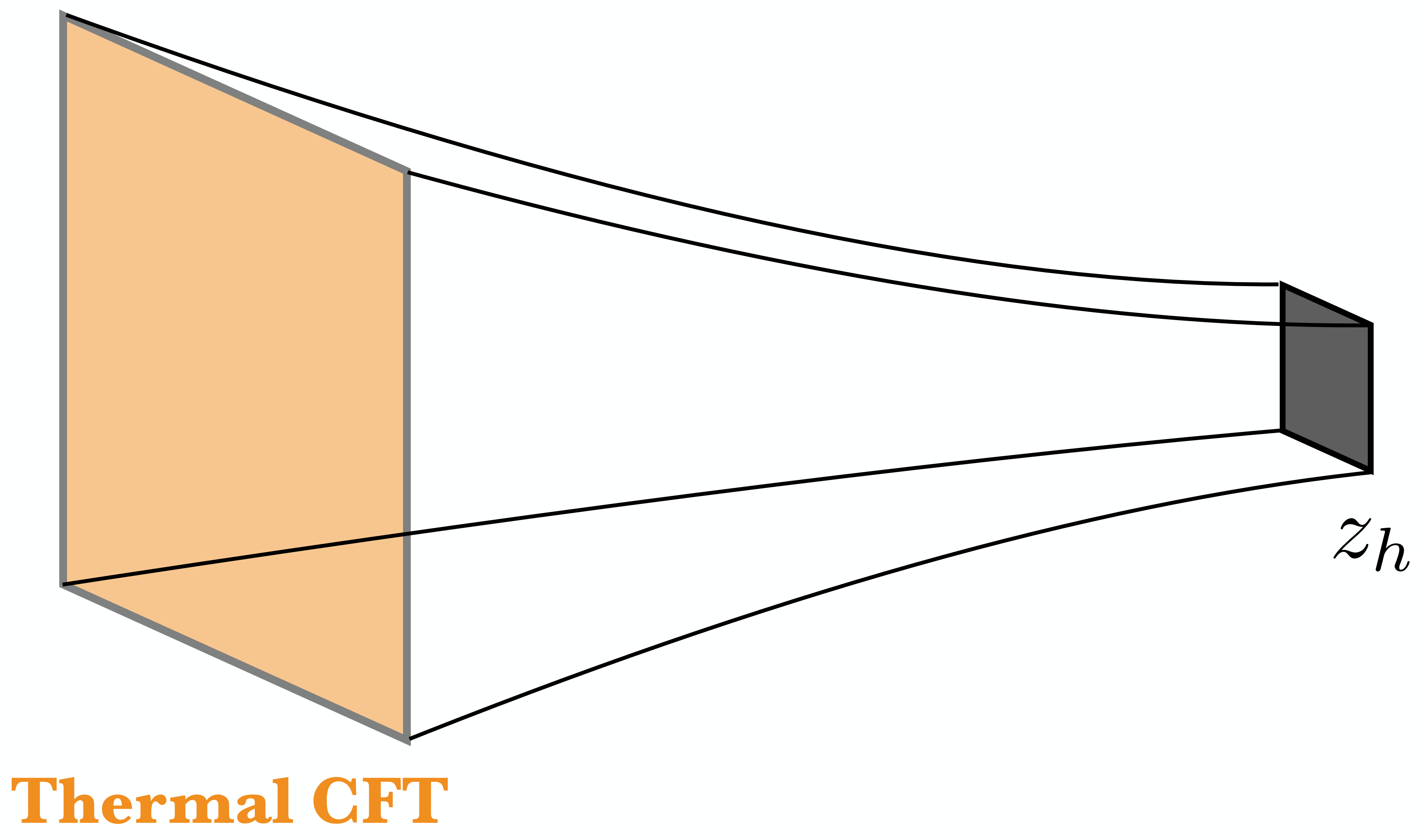
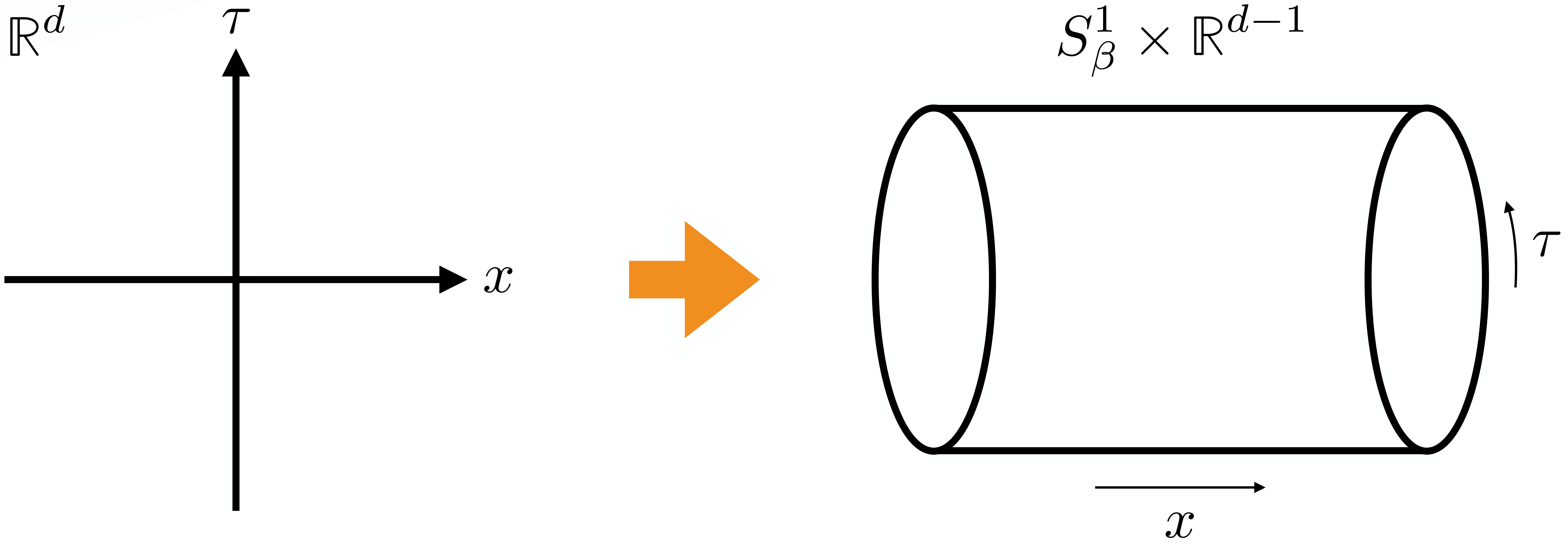


Thermal CFTs are dual to AdS black holes



1. CFT at finite temperature

FROM CFT TO THERMAL CFT: GEOMETRY



$$\beta = \frac{1}{T}$$

$$x = |\vec{x}|$$

1. CFT at finite temperature

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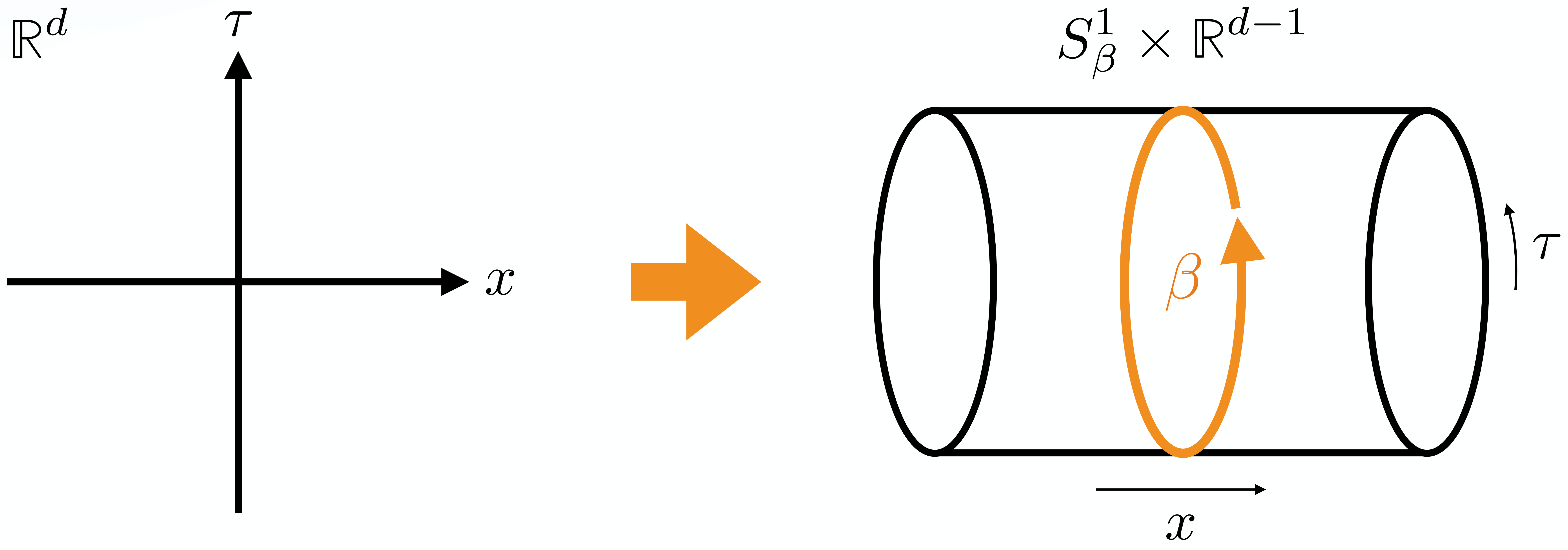

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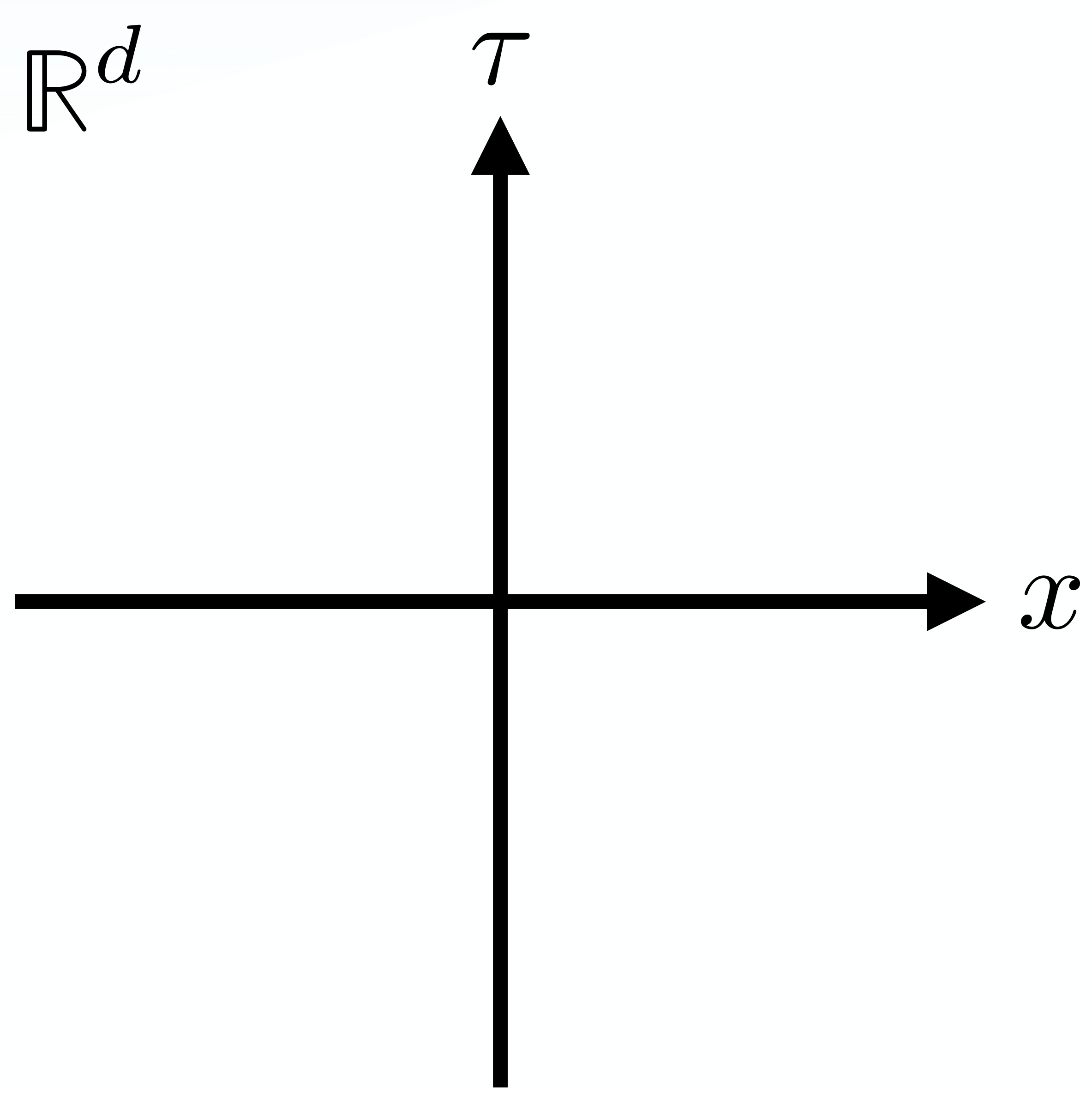

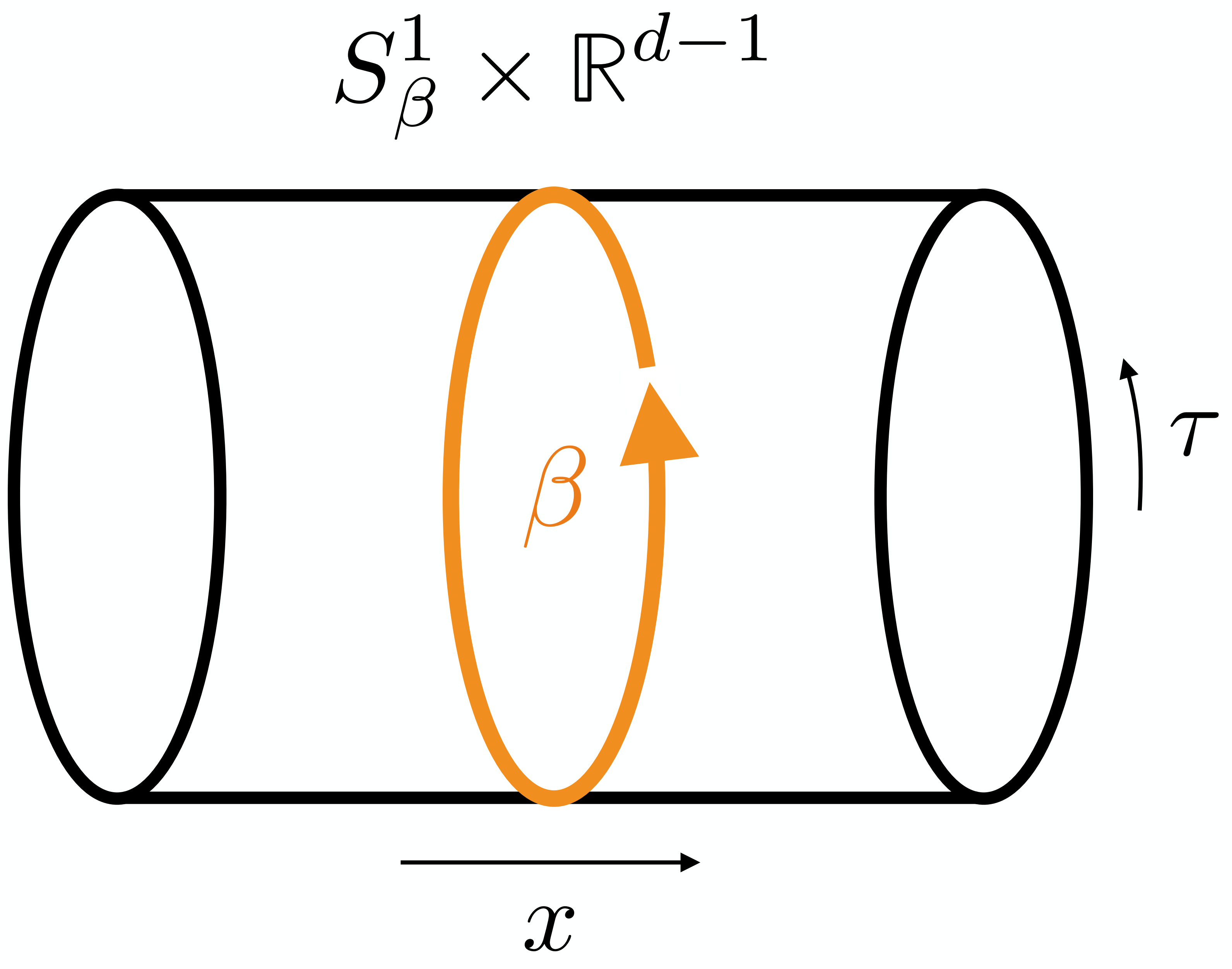

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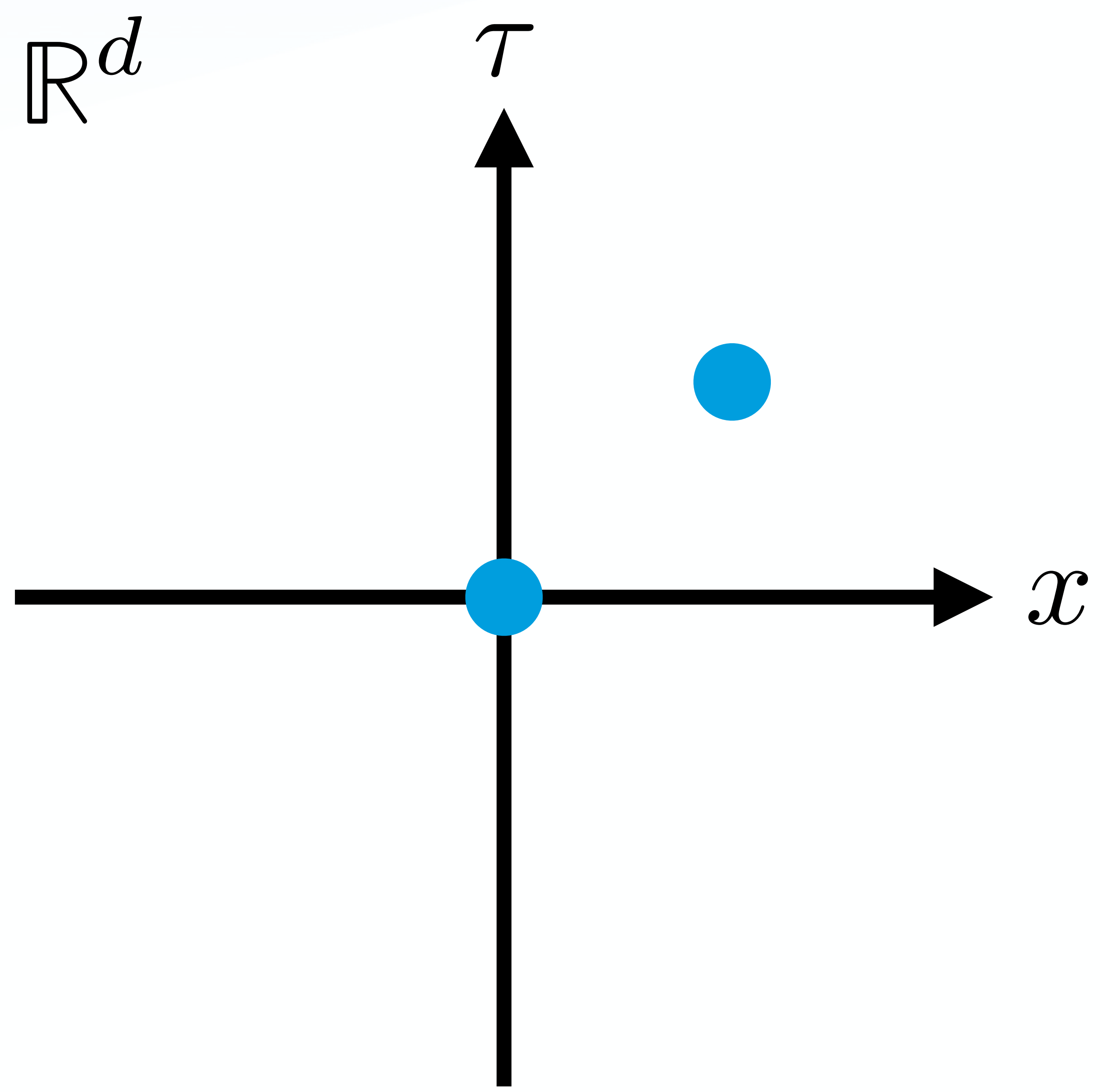
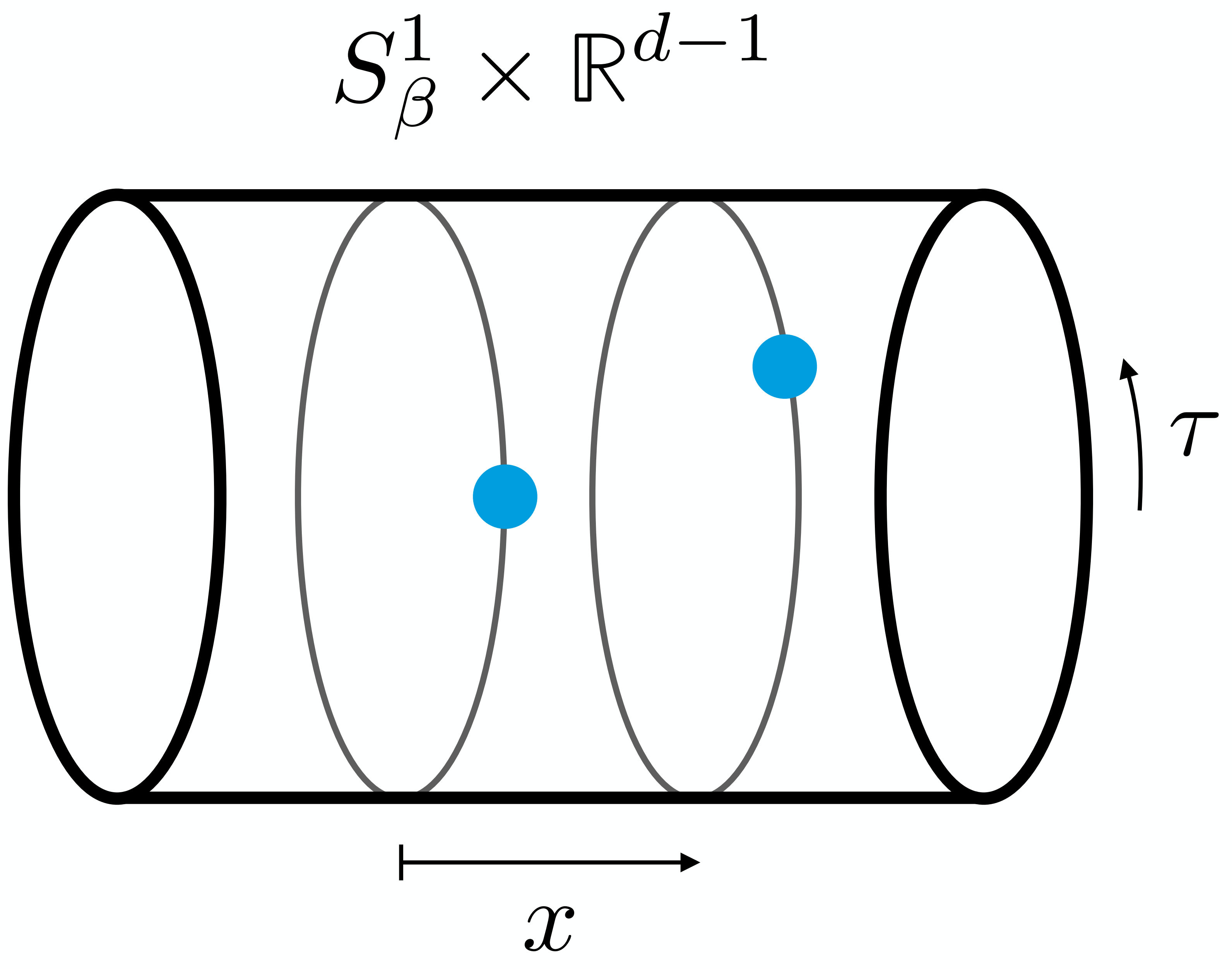
New scale β breaks conformal symmetry

1. CFT at finite temperature

FROM CFT TO THERMAL CFT: GEOMETRY





$$\beta = \frac{1}{T}$$

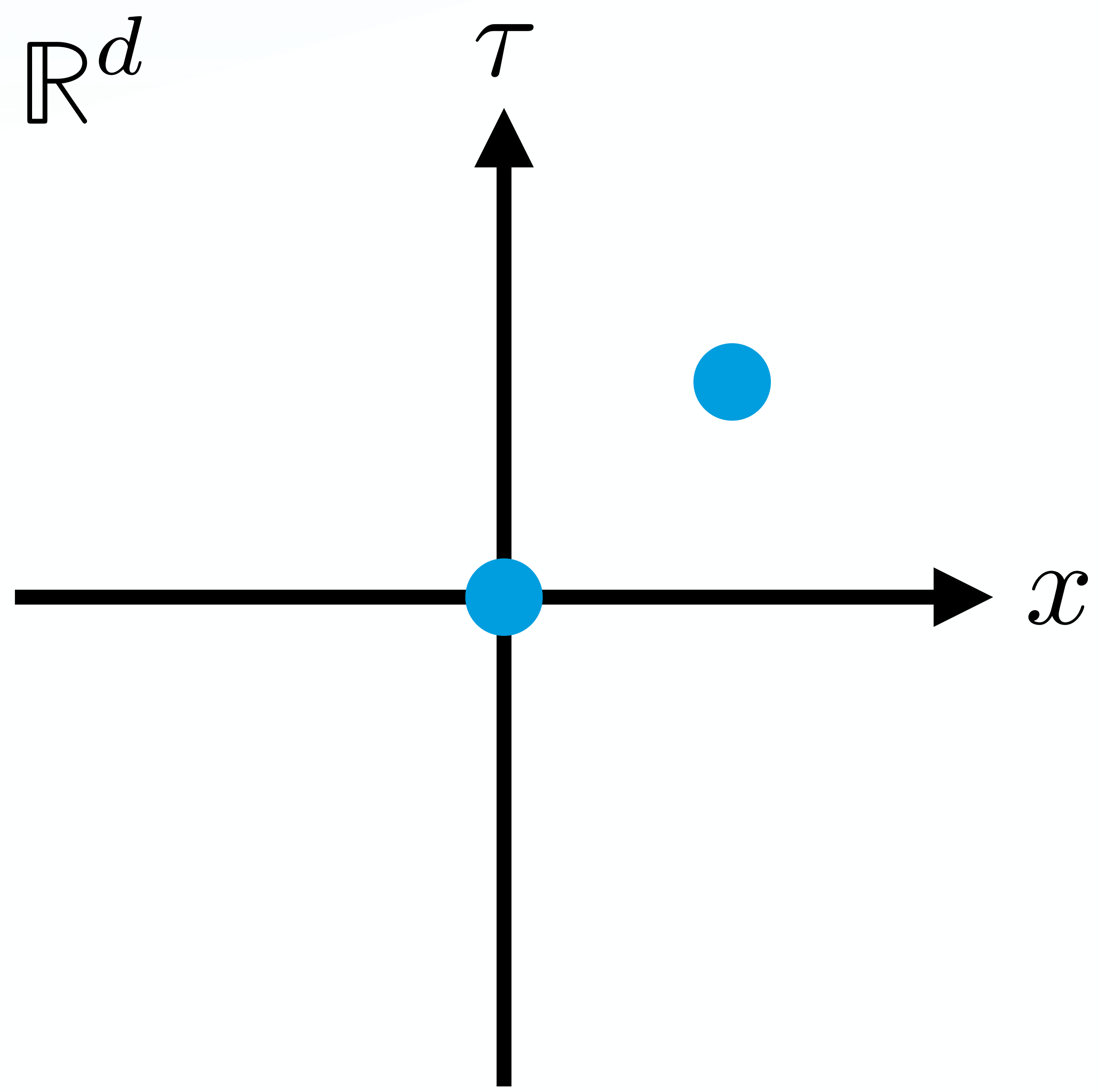
$$x = |\vec{x}|$$

$$\langle\, \phi(0,0)\phi(\tau,x)\,\rangle$$

$$\langle\, \phi(0,0)\phi(\tau,x)\,\rangle_\beta$$

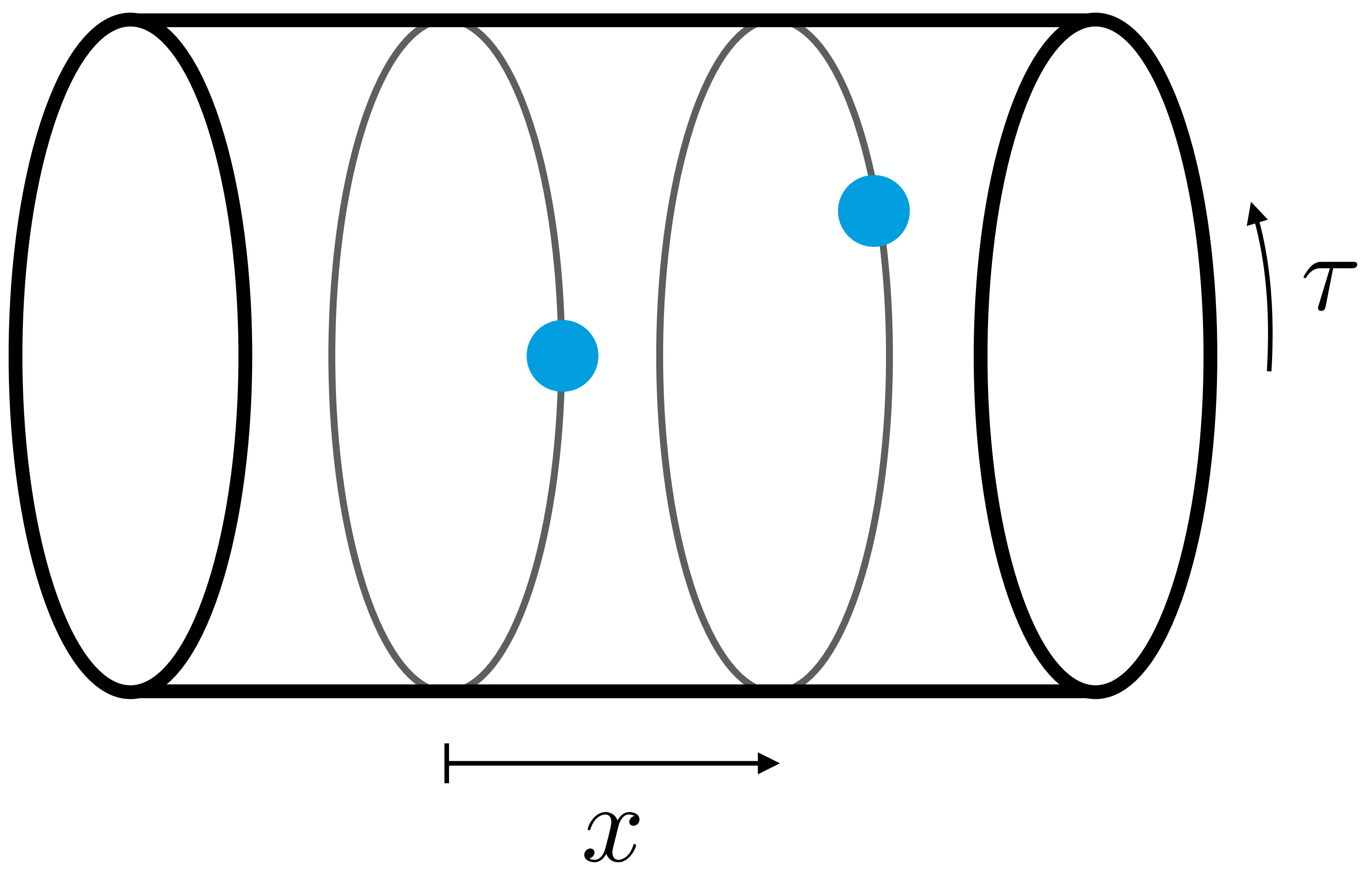
1. CFT at finite temperature

FROM CFT TO THERMAL CFT: GEOMETRY



$$\langle\, \phi(0,0)\phi(\tau,x)\,\rangle$$

$S^1_\beta \times \mathbb{R}^{d-1}$



$$\beta = \frac{1}{T}$$

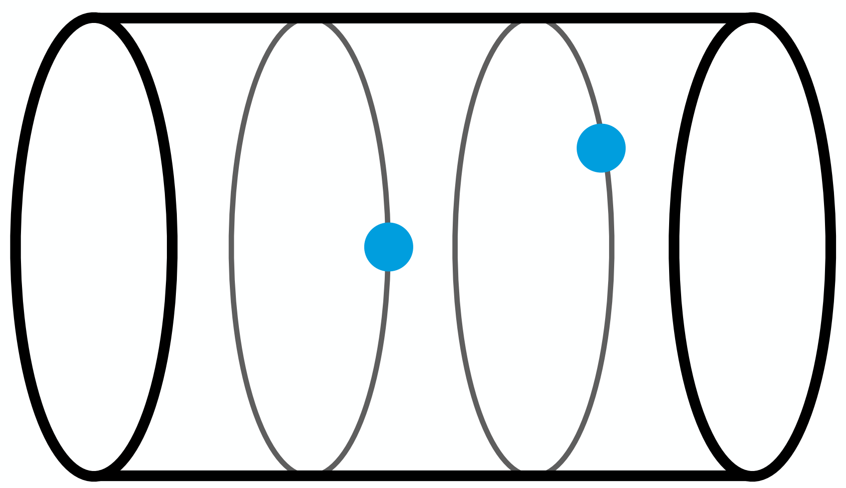
$$x = |\vec{x}|$$

$$\boxed{\langle\, \phi(0,0)\phi(\tau,x)\,\rangle_\beta}$$

Central objects of this talk

1. CFT at finite temperature

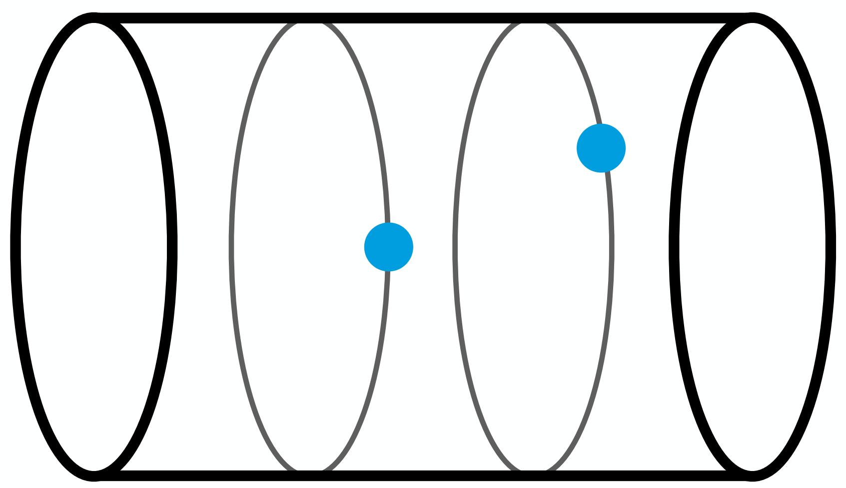
FROM CFT TO THERMAL CFT: KINEMATICS



1. CFT at finite temperature

FROM CFT TO THERMAL CFT: KINEMATICS

	CFT	Thermal CFT
$\langle\, \phi(0,0)\phi(\tau,x)\,\rangle$	$\dfrac{1}{(\tau^2+x^2)^{\Delta_\phi}}$	$g(\tau,x)$
OPE	converges everywhere	$\tau^2+x^2<\beta^2$
CFT data	$\Delta_{\mathcal{O}}\,,\, f_{\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k}$	$b_{\mathcal{O}}$



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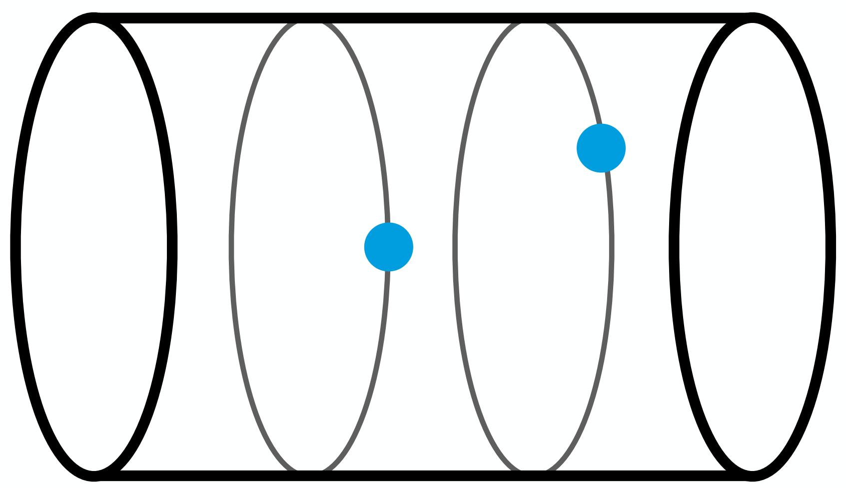

Thermal one-point functions: $\langle\, \mathcal{O}_{\mu_1\ldots\mu_J}(x)\,\rangle_\beta = \dfrac{b_{\mathcal{O}}}{\beta^\Delta}(\delta_{\mu_1 0}\ldots\delta_{\mu_J 0} - \text{traces})$

1. CFT at finite temperature

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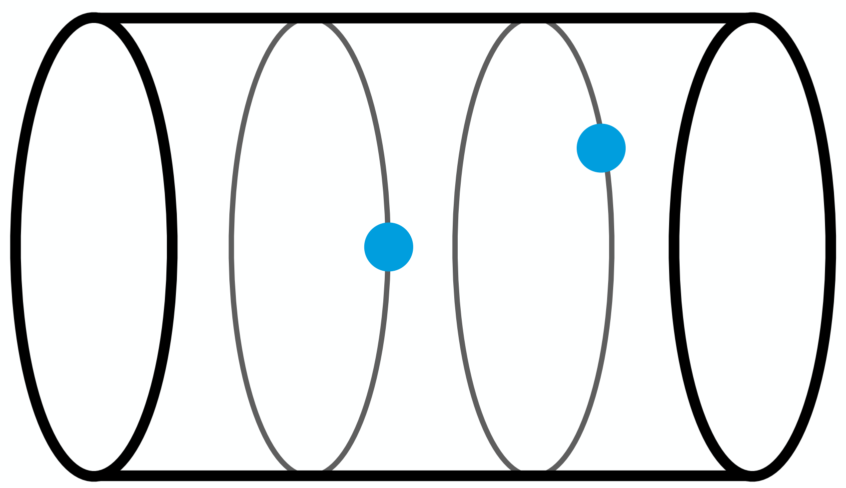

$\rightarrow$ **New unknowns**

Thermal one-point functions: $\langle\, \mathcal{O}_{\mu_1\ldots\mu_J}(x)\,\rangle_\beta = \dfrac{\boxed{b_{\mathcal{O}}}}{\beta^\Delta}(\delta_{\mu_1 0}\ldots\delta_{\mu_J 0} - \text{traces})$

1. CFT at finite temperature

FROM CFT TO THERMAL CFT: KINEMATICS

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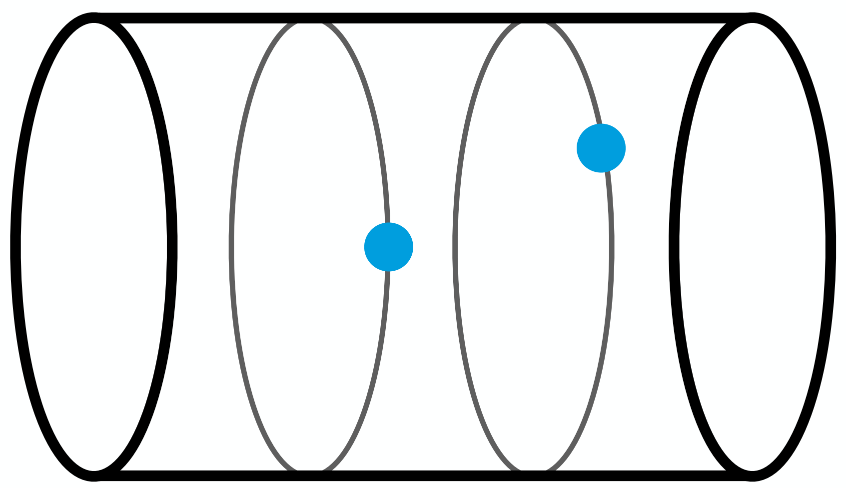

$\longrightarrow$ **New unknowns**

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The spectrum **stays the same**.

1. CFT at finite temperature

DYNAMICAL INFORMATION

We will focus on **two configurations**:

1. CFT at finite temperature

DYNAMICAL INFORMATION

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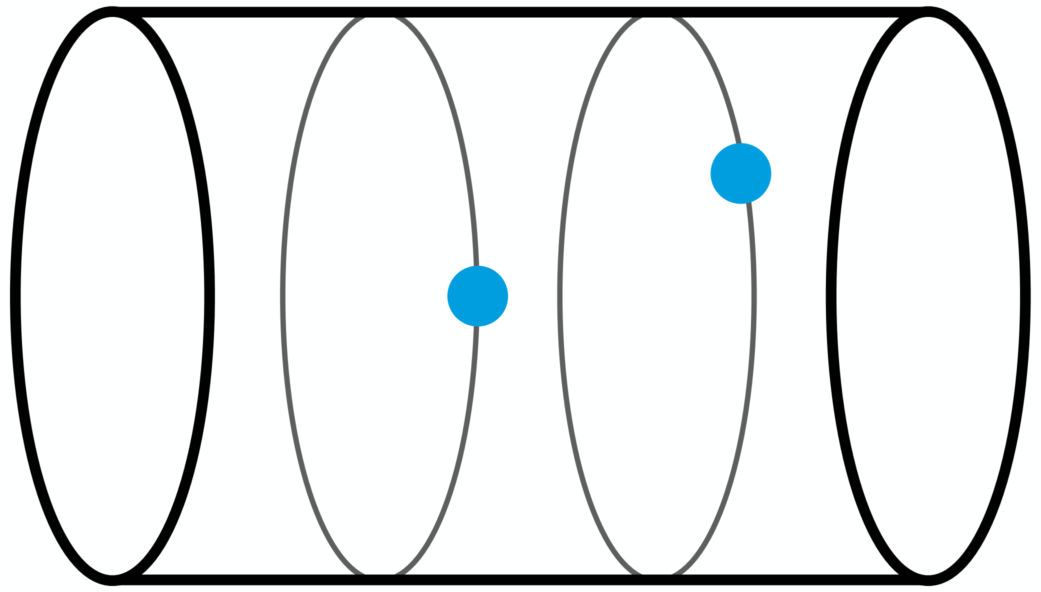
$g(\tau, x)$ $\qquad\qquad$ $g(\tau)$

DYNAMICAL INFORMATION

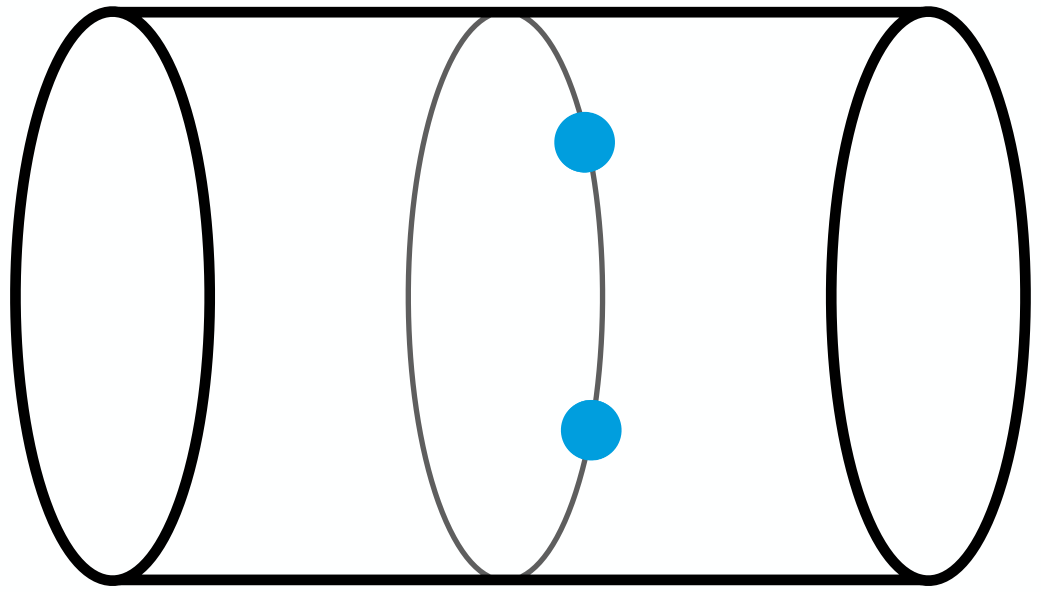
We will focus on **two configurations**:

$g(\tau, x)$

“Normal"



$g(\tau)$



“Easy”

DYNAMICAL INFORMATION

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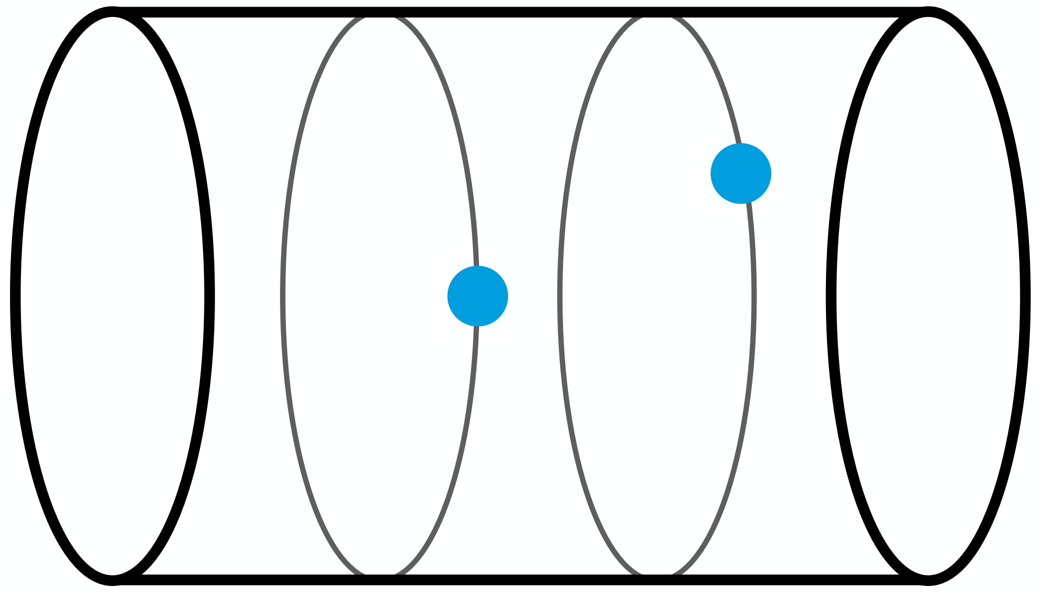
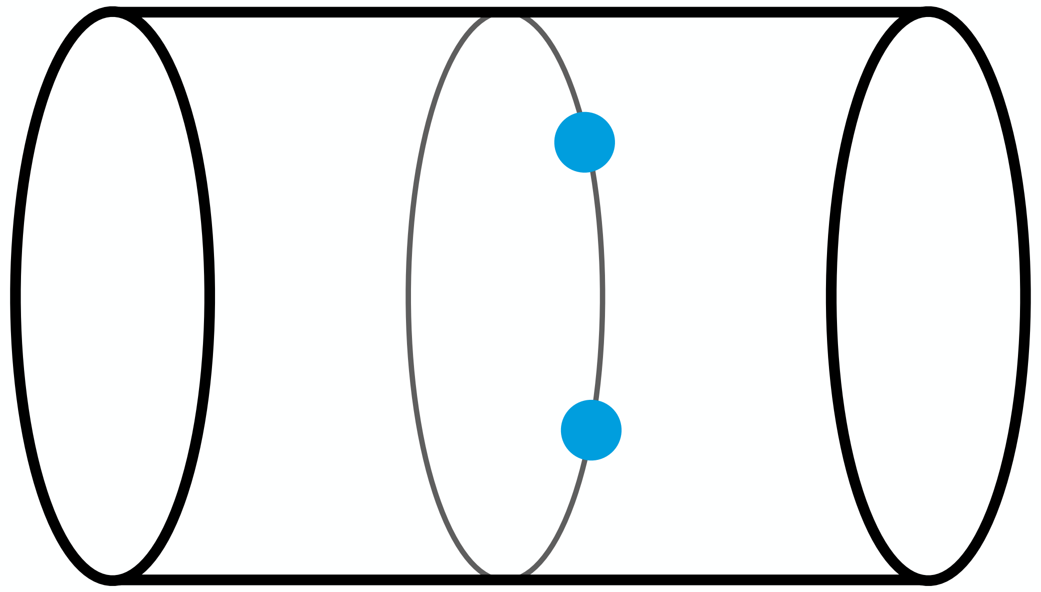


$$g(\tau, x) = \sum_{\mathcal{O}} a_{\mathcal{O}}\, f_{\Delta,J}(\tau, x)$$

$g(\tau)$



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1. CFT at finite temperature

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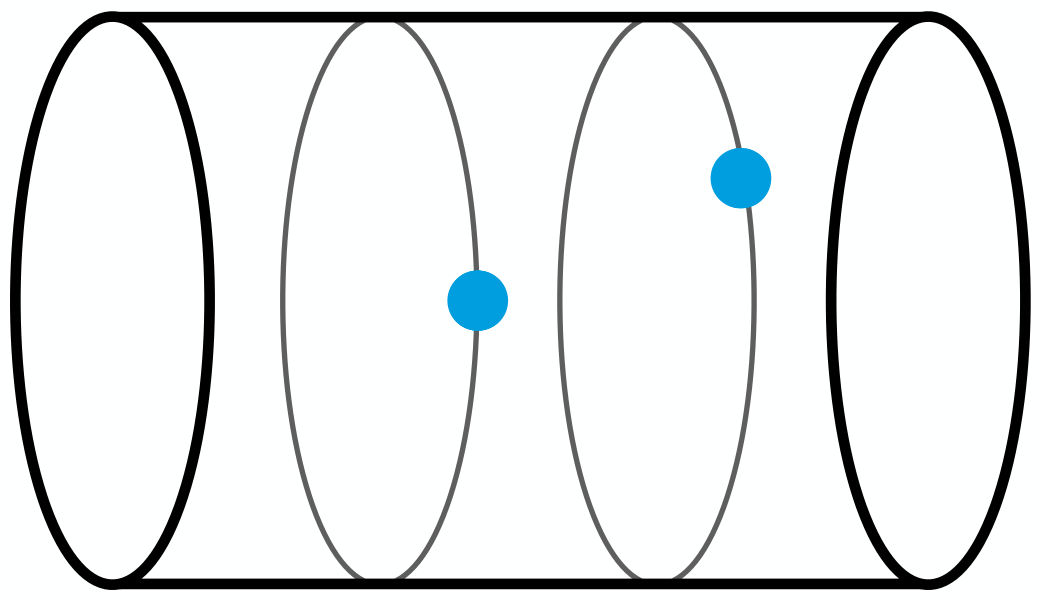

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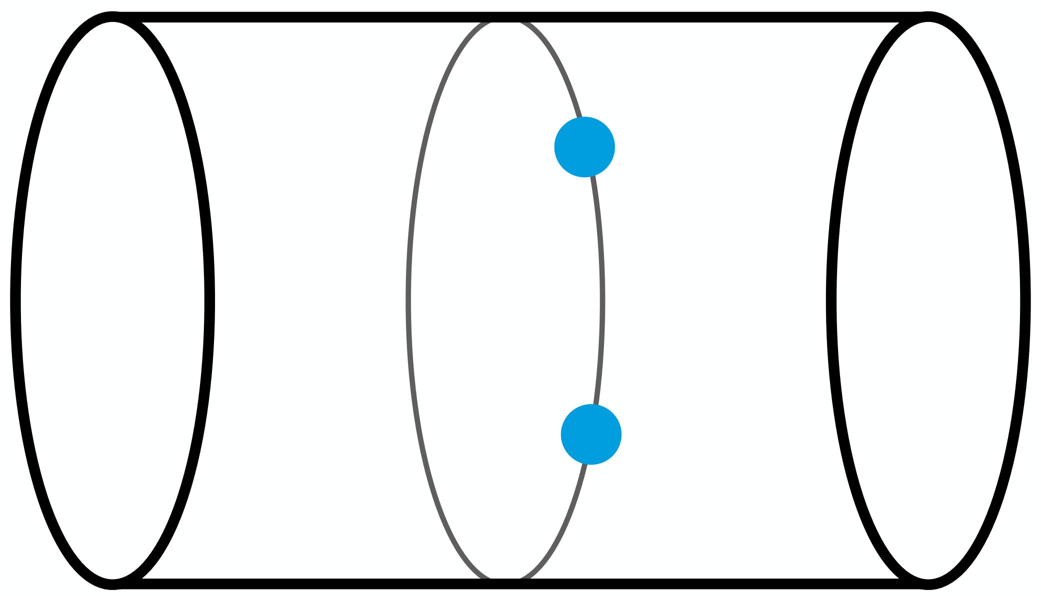

"Easy"

$$g(\tau, x) = \sum_{\mathcal{O}} a_{\mathcal{O}} \boxed{f_{\Delta,J}(\tau, x)}$$

Thermal conformal blocks

1. CFT at finite temperature

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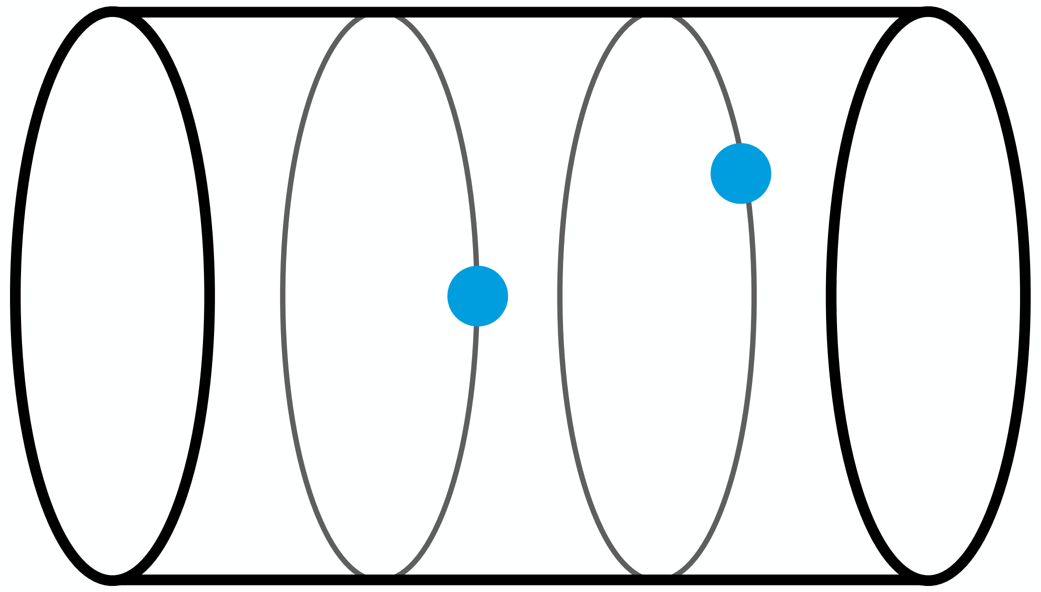

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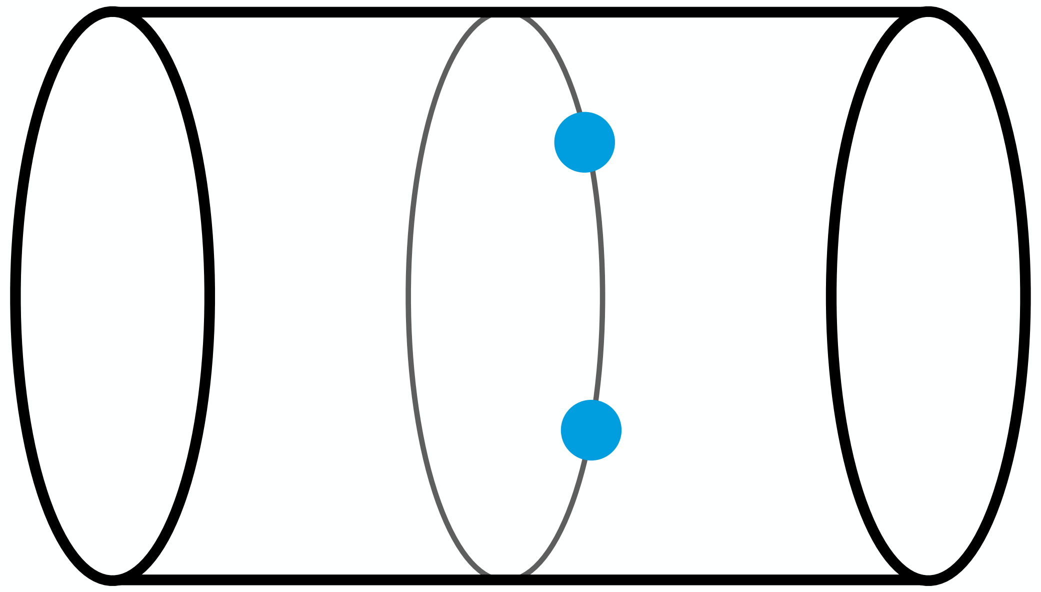

“Easy”

$$g(\tau, x) = \sum_{\mathcal{O}} \boxed{a_{\mathcal{O}}} f_{\Delta,J}(\tau, x)$$

Thermal OPE coefficients

$$a_{\mathcal{O}} \sim f_{\phi\phi\mathcal{O}} b_{\mathcal{O}}$$

1. CFT at finite temperature

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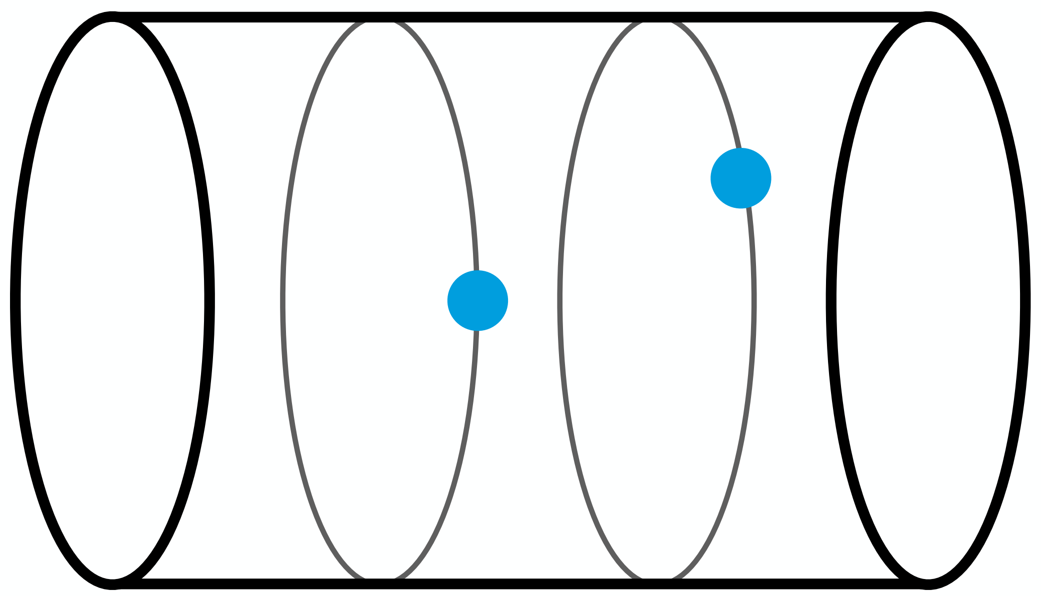

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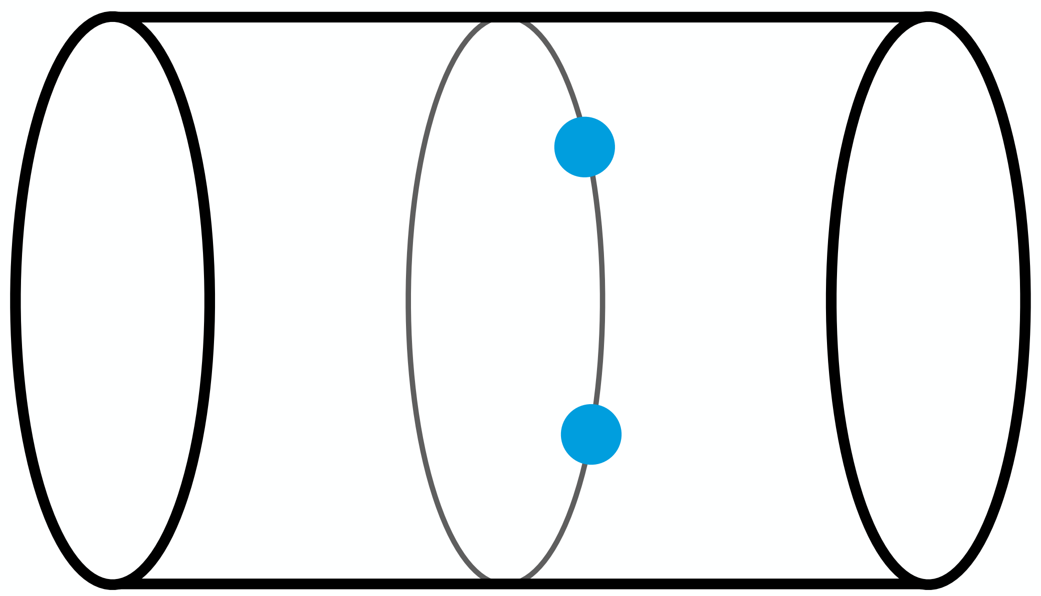

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3-point functions at $T = 0$

1. CFT at finite temperature

DYNAMICAL INFORMATION

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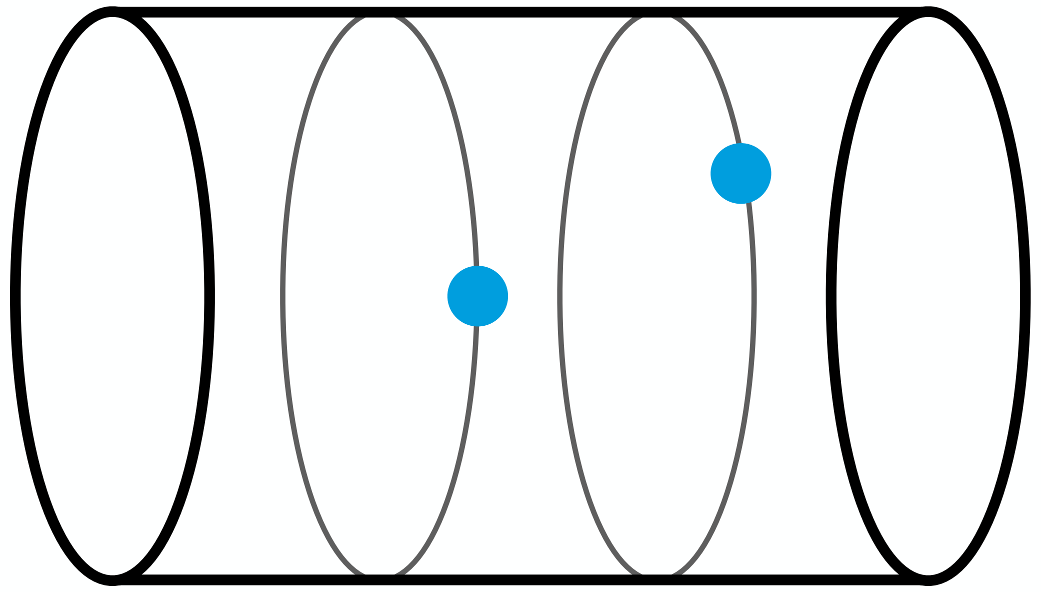
$g(\tau, x)$

“Normal"



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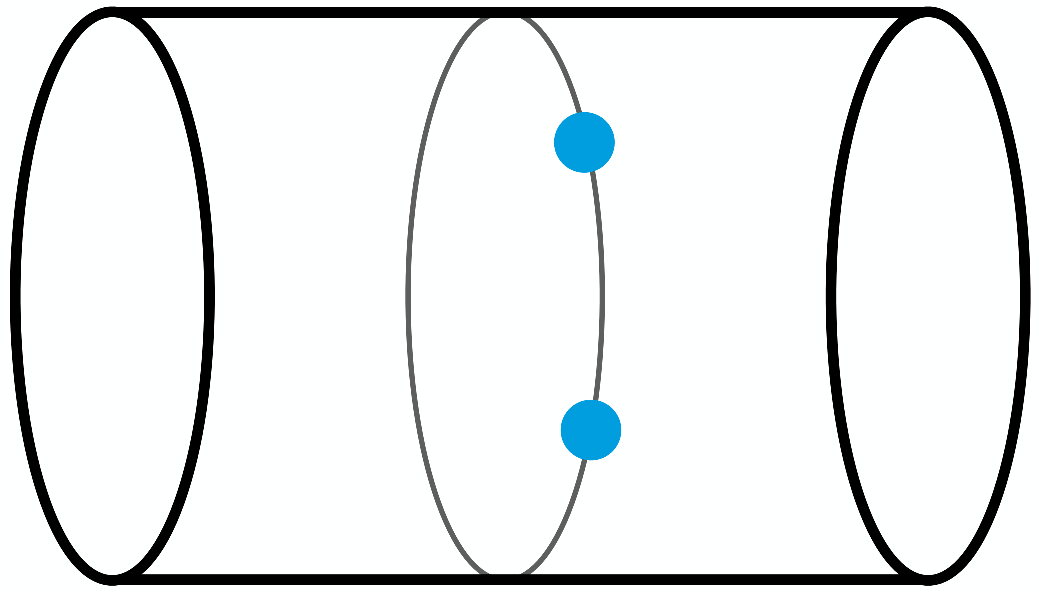

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3-point functions at $T = 0$ **Thermal 1-point functions**

1. CFT at finite temperature

DYNAMICAL INFORMATION

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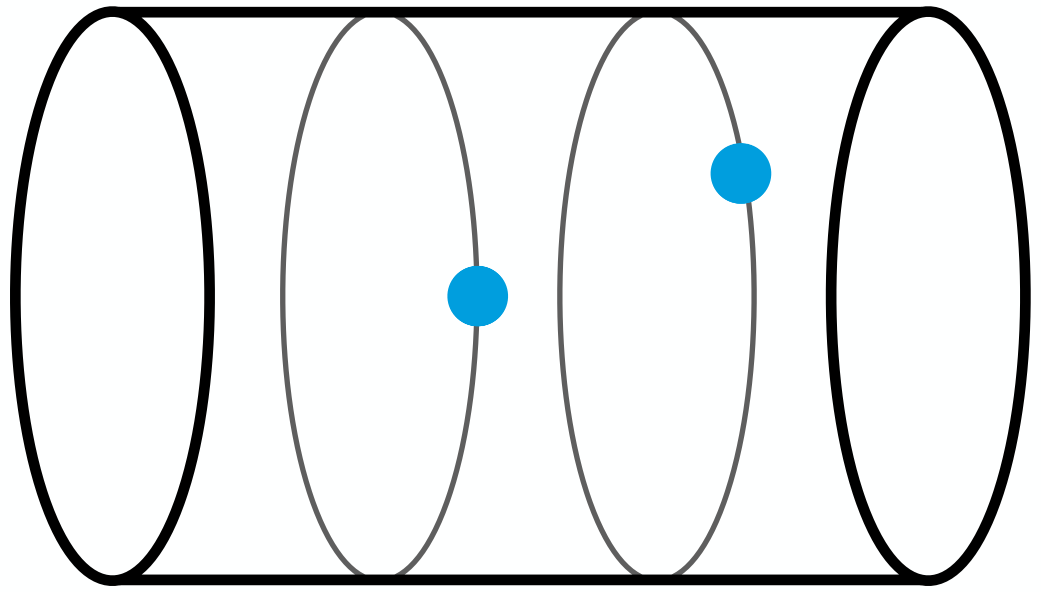

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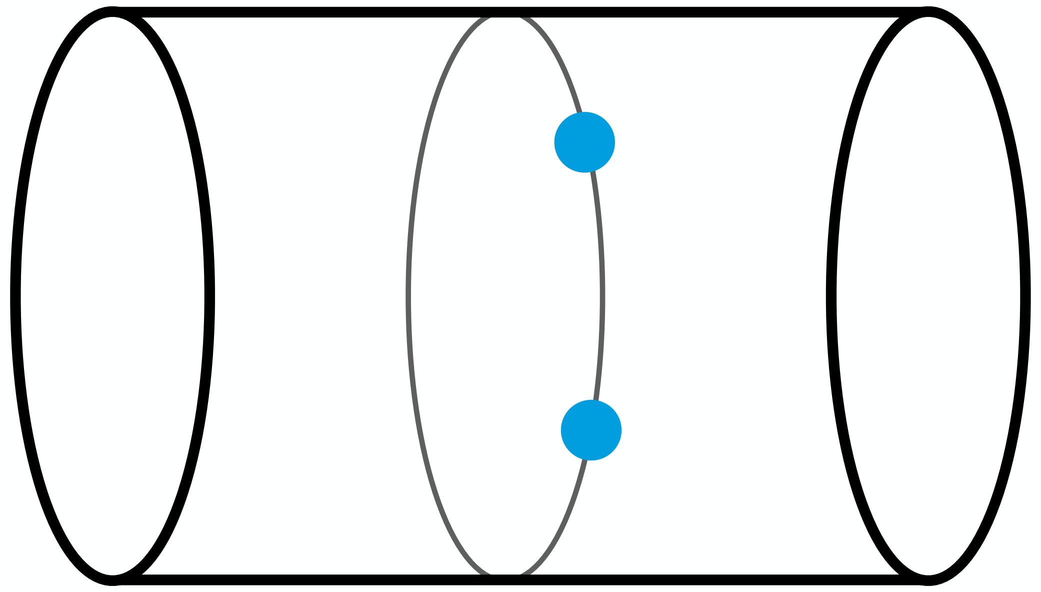
Thermal OPE coefficients

$$a_{\mathcal{O}} \sim \underbrace{f_{\phi\phi\mathcal{O}}}_{\text{3-point functions at } T=0} \underbrace{b_{\mathcal{O}}}_{\text{Thermal 1-point functions}}$$

3-point functions at $T = 0$ **Thermal 1-point functions**

$g(\tau)$



“Easy”

$$g(\tau) = \sum_{\Delta} a_{\Delta} \left(\frac{\tau}{\beta}\right)^{\Delta - 2\Delta_\phi}$$

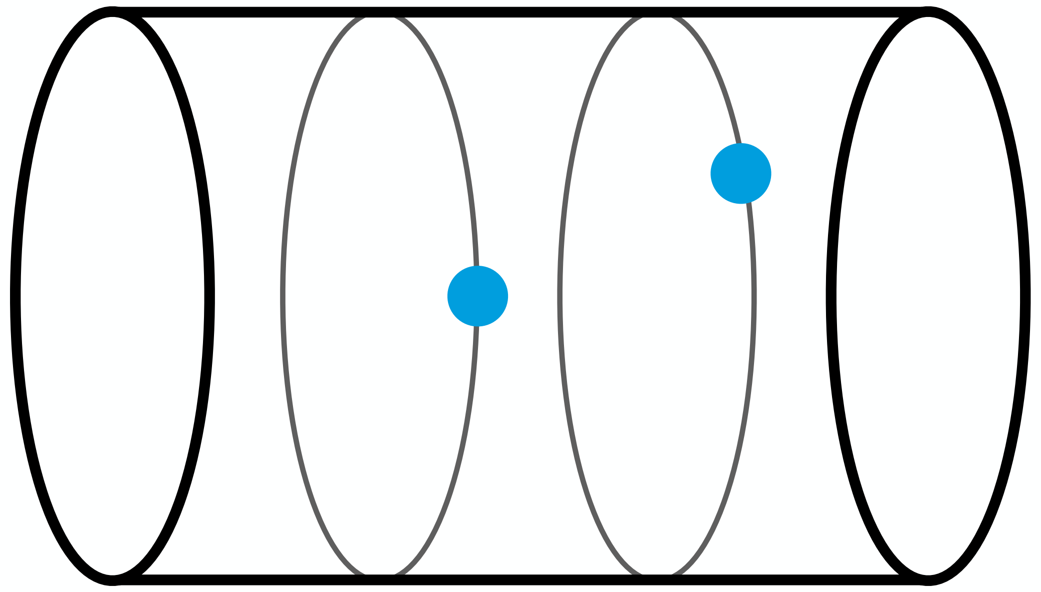
1. CFT at finite temperature

DYNAMICAL INFORMATION

We will focus on **two configurations**:

$g(\tau, x)$

"Normal"



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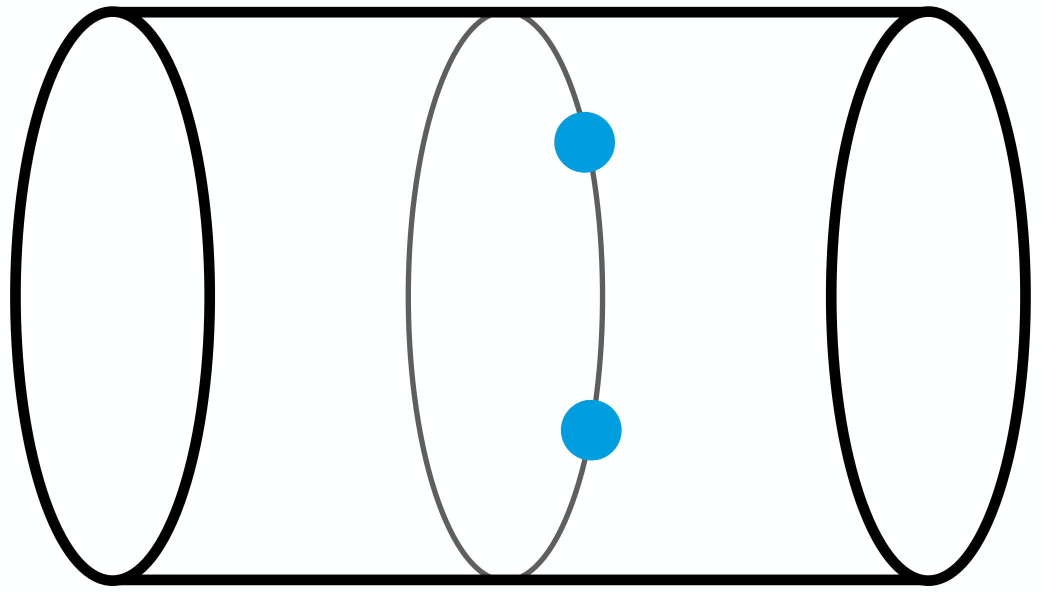
3-point functions at $T = 0$ **Thermal 1-point functions**

$g(\tau)$



"Easy"

$$g(\tau) = \sum_{\Delta} \boxed{a_{\Delta}} \left(\frac{\tau}{\beta}\right)^{\Delta - 2\Delta_\phi}$$

$$a_\Delta = \sum_{\Delta_{\mathcal{O}} = \Delta} a_{\mathcal{O}} C_J^{(d/2-1)}(1)$$

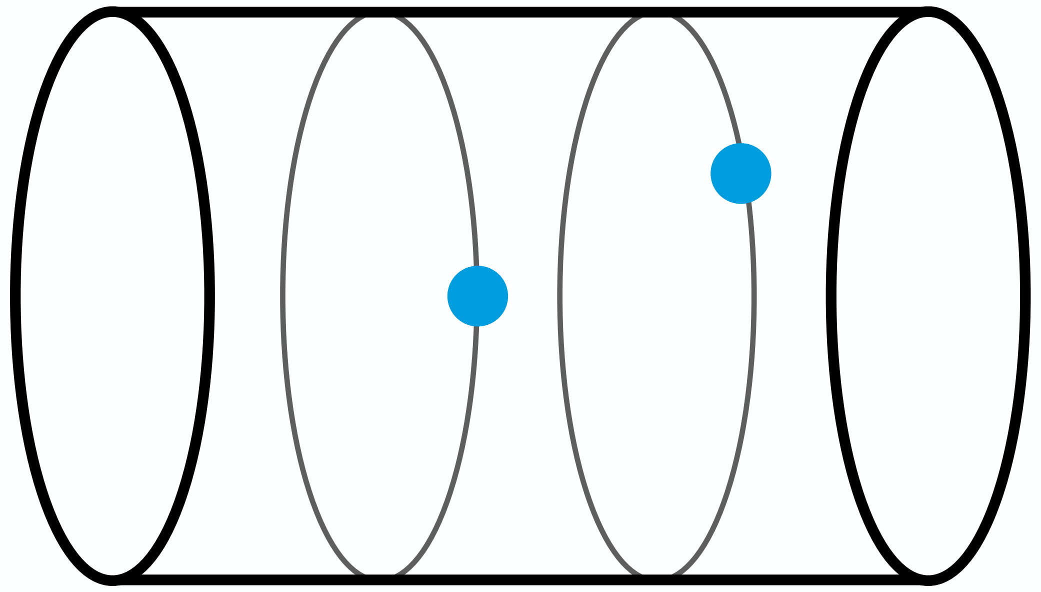
1. CFT at finite temperature

DYNAMICAL INFORMATION

We will focus on **two configurations**:

$g(\tau, x)$

"Normal"



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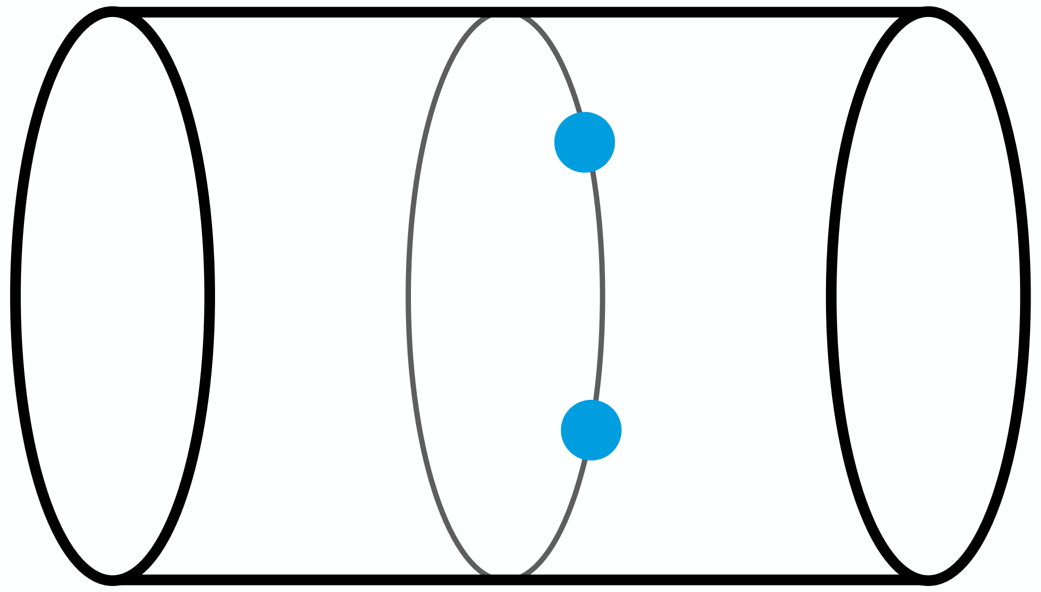

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3-point functions at $T = 0$ **Thermal 1-point functions**

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(OPE converges everywhere)

1. CFT at finite temperature

PERIODICITY AS A BOOTSTRAP CONSTRAINT

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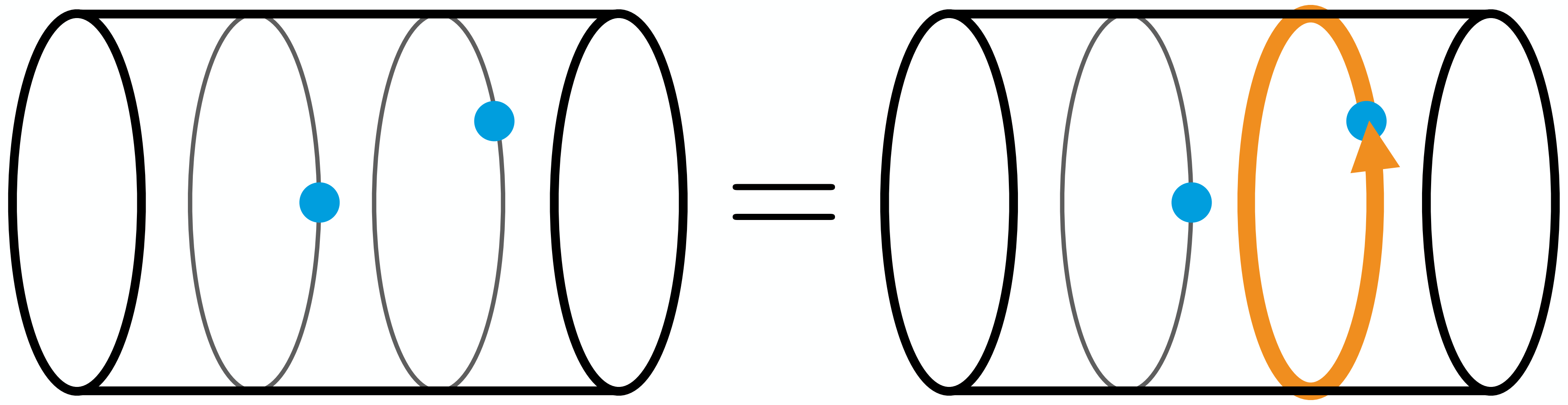
Periodicity: $$g(\tau, x) = g(\tau + \beta, x)$$

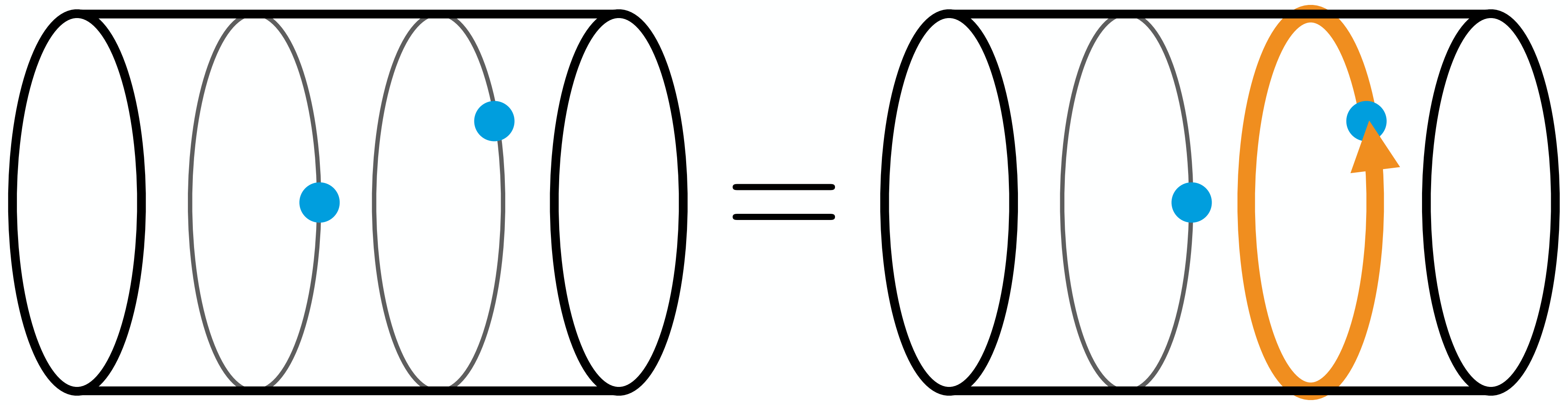


[Kubo, '57] [Martin, Schwinger, '59]

1. CFT at finite temperature

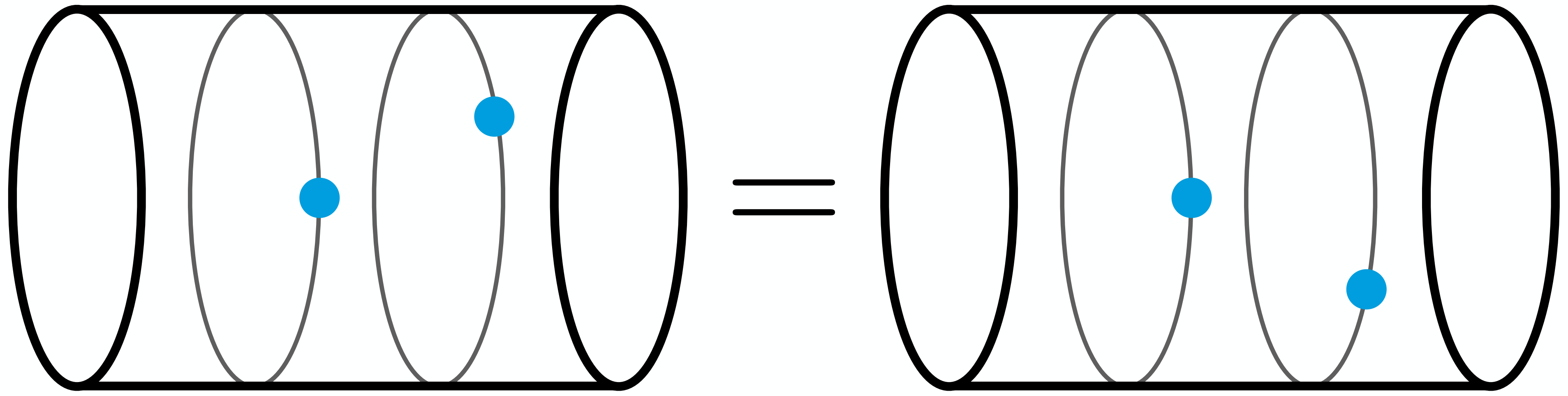
PERIODICITY AS A BOOTSTRAP CONSTRAINT

Periodicity: $g(\tau, x) = g(\tau + \beta, x)$



[Kubo, '57] [Martin, Schwinger, '59]

Periodicity + parity: $g(\tau, x) = g(\beta - \tau, x)$

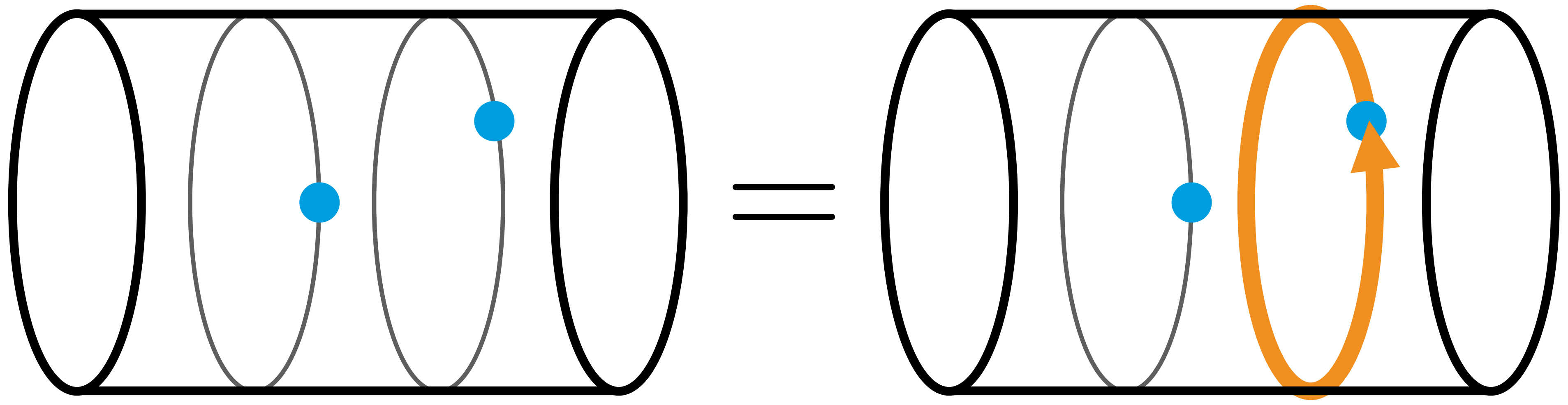


[El-Showk, Papadodimas, '11]

1. CFT at finite temperature

PERIODICITY AS A BOOTSTRAP CONSTRAINT

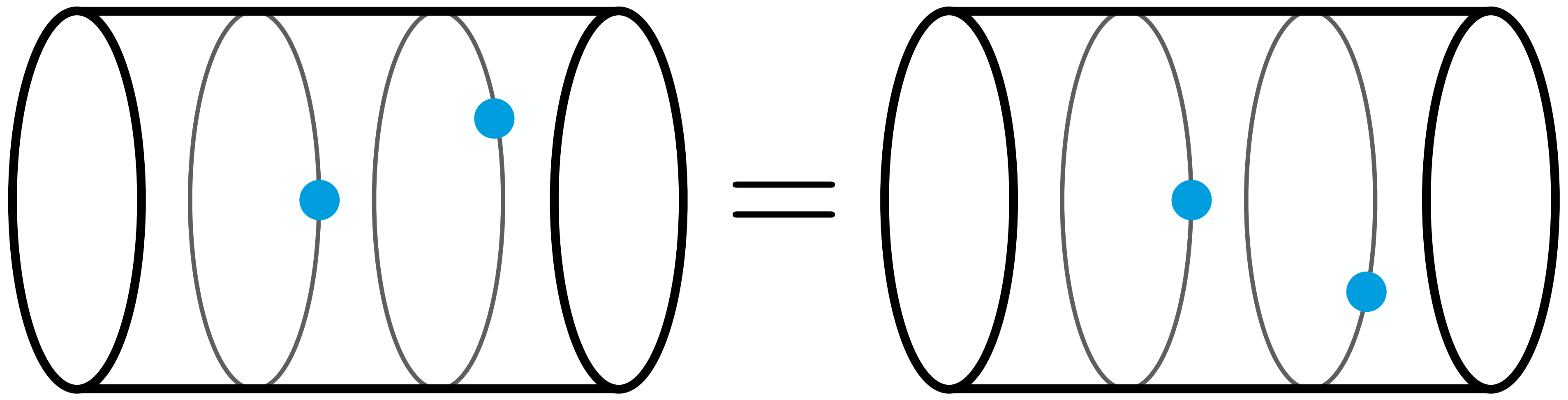
Periodicity: $g(\tau, x) = g(\tau + 1, x)$



[Kubo, '57] [Martin, Schwinger, '59]

$(\beta = 1$ from now on$)$

Periodicity + parity: $g(\tau, x) = g(1 - \tau, x)$



[El-Showk, Papadodimas, '11]

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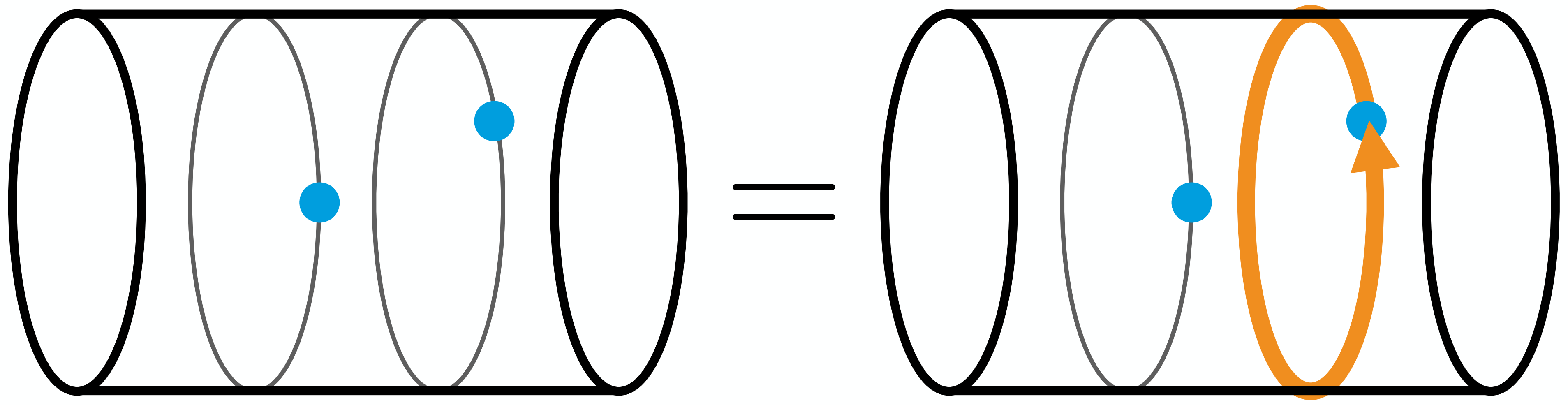

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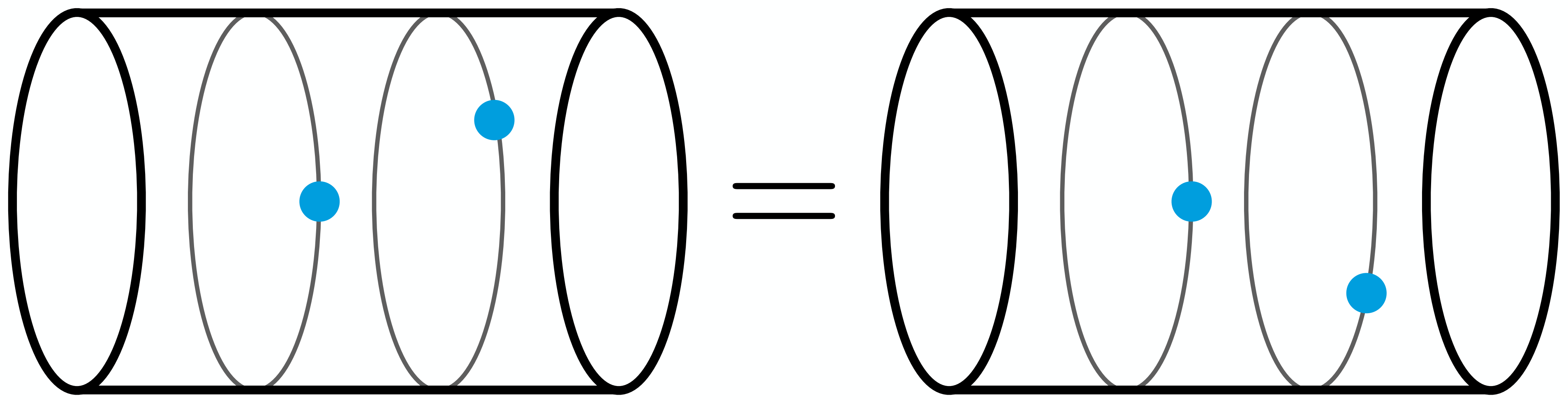

[El-Showk, Papadodimas, '11]

- Periodicity plays the role of a **crossing equation**;

1. CFT at finite temperature

PERIODICITY AS A BOOTSTRAP CONSTRAINT

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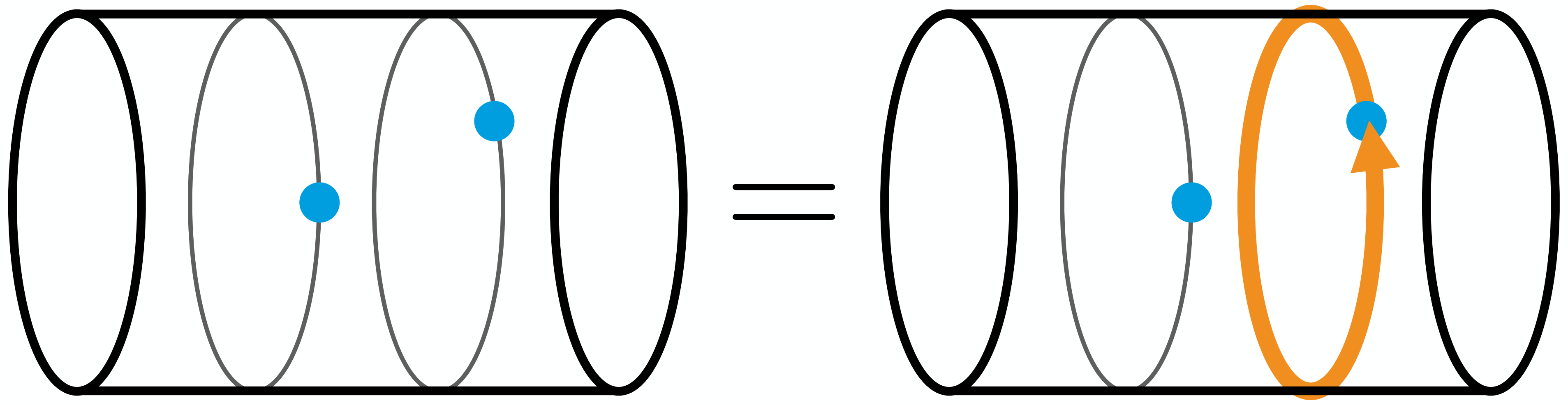

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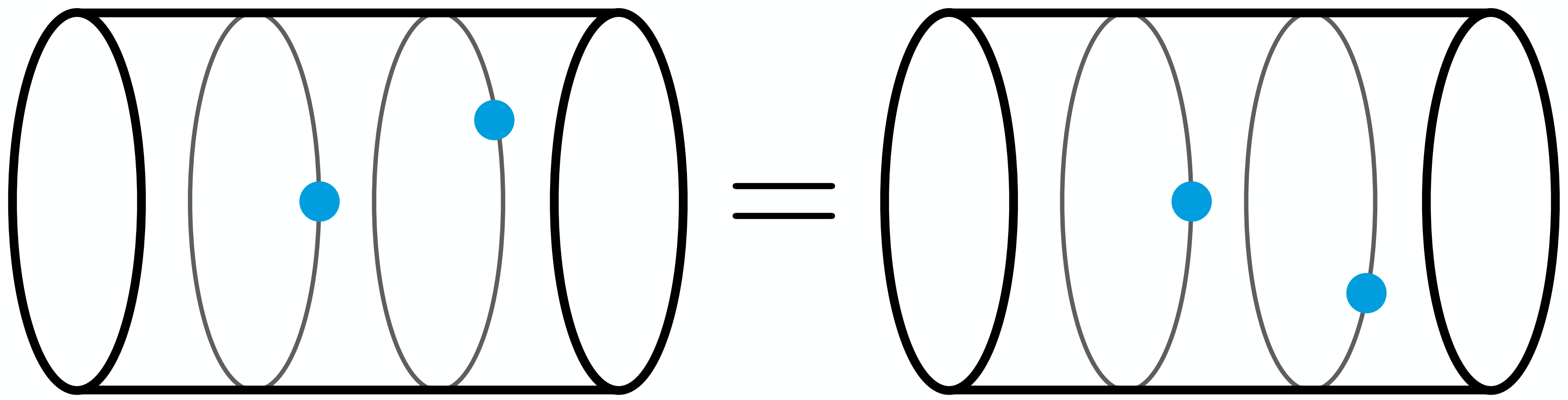
Periodicity + parity: $g(\tau, x) = g(1 - \tau, x)$

***s*-channel** **t*-channel**



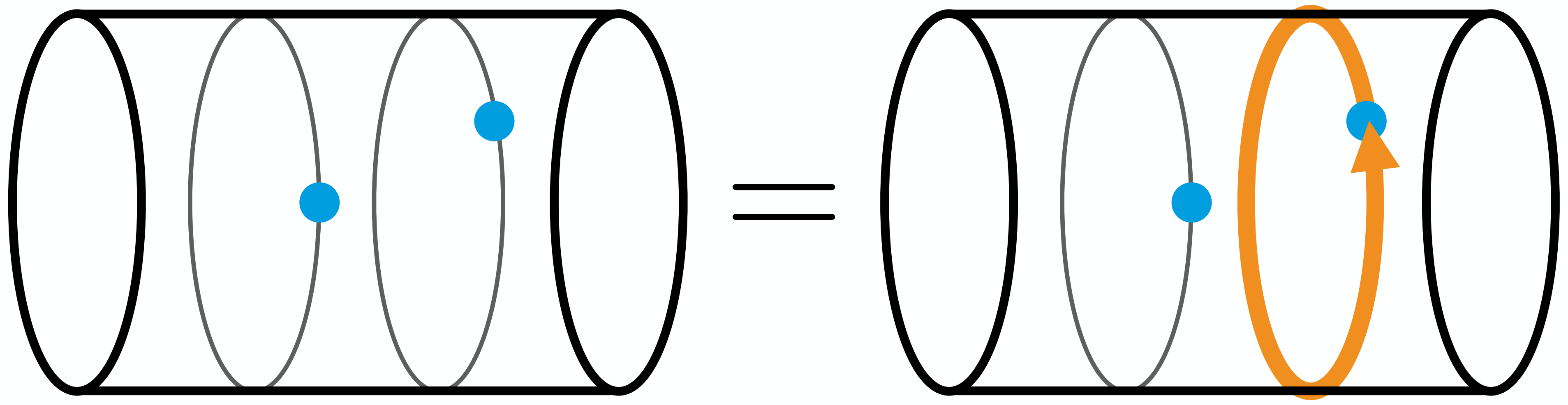
[El-Showk, Papadodimas, '11]

- Periodicity plays the role of a **crossing equation**;

1. CFT at finite temperature

PERIODICITY AS A BOOTSTRAP CONSTRAINT

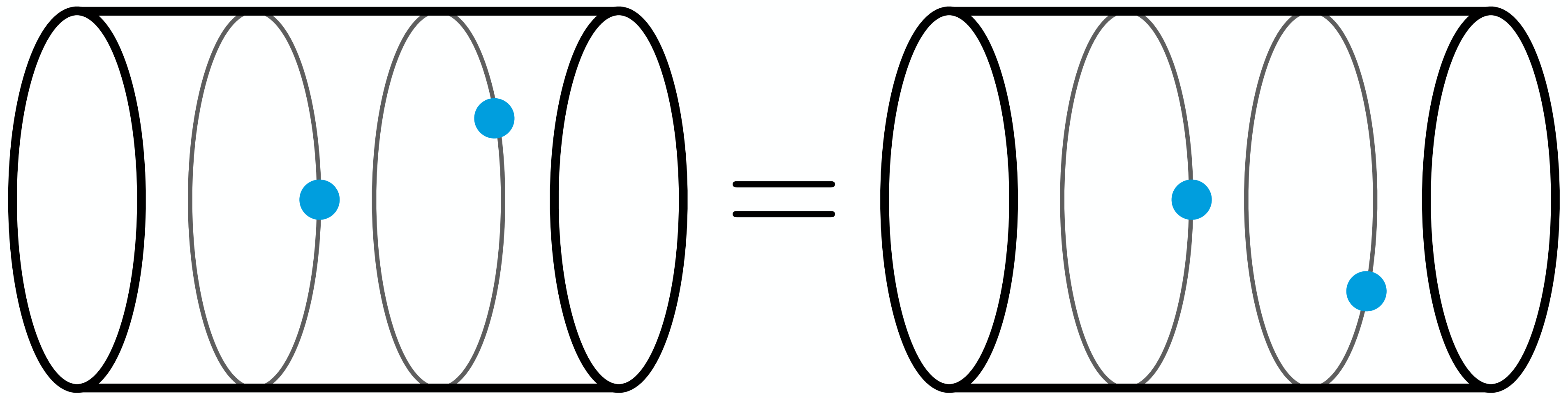
Periodicity: $g(\tau, x) = g(\tau + 1, x)$



[Kubo, '57] [Martin, Schwinger, '59]

$(\beta = 1$ from now on$)$

Periodicity + parity: $\underbrace{g(\tau, x)}_{s\text{-channel}} = \underbrace{g(1 - \tau, x)}_{t\text{-channel}}$



[El-Showk, Papadodimas, '11]

- Periodicity plays the role of a **crossing equation**;
- The OPE is **not periodic**.

1. CFT at finite temperature

AN EXAMPLE: GFF

EOM: $\Box^{d/2-\Delta_\phi}\phi = 0$

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Method of images

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$\Delta_\phi = 1$: $g(\tau) = \pi^2 \csc^2(\pi\tau)$

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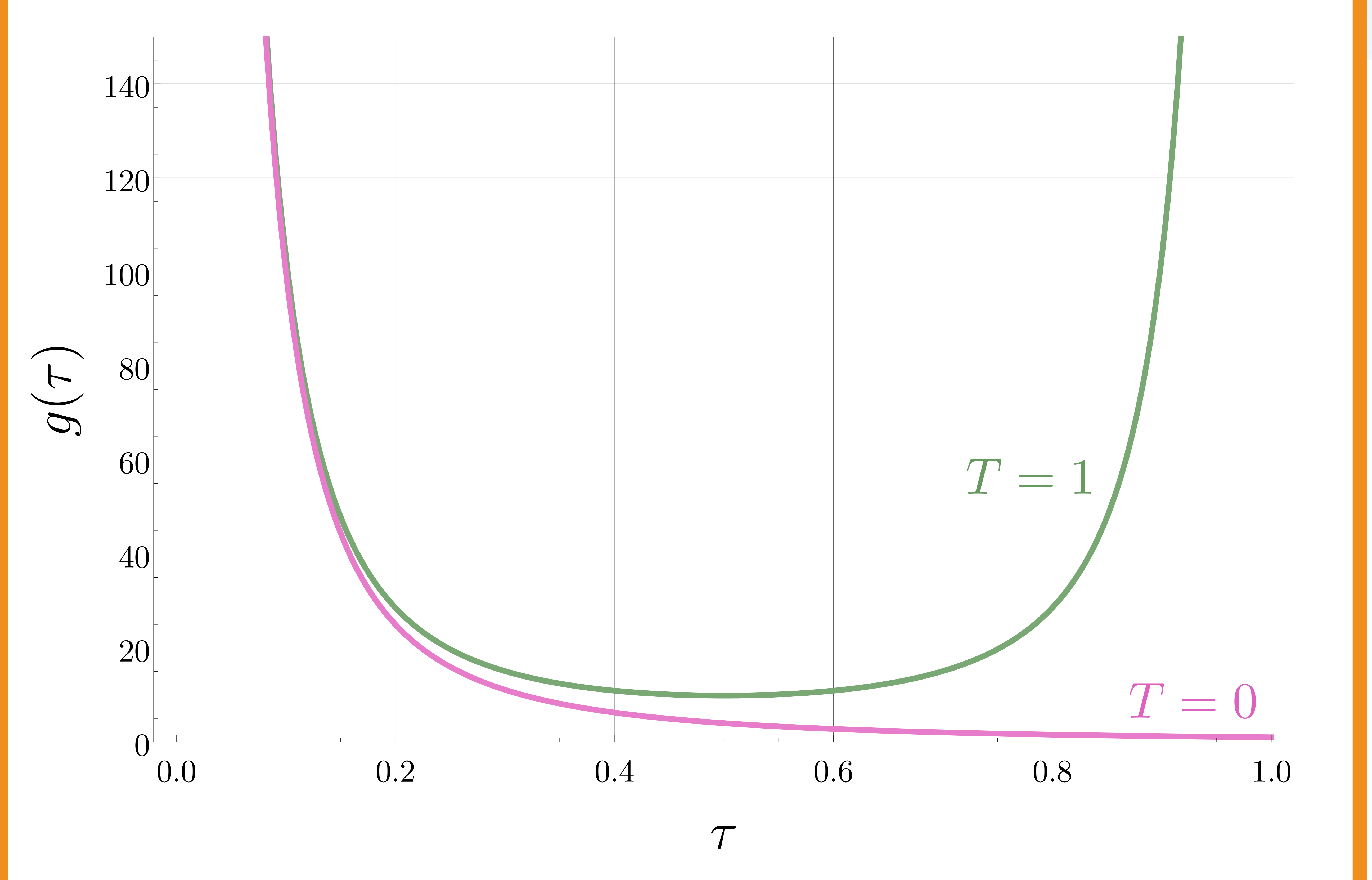

2. Bootstrapping thermal CFTs

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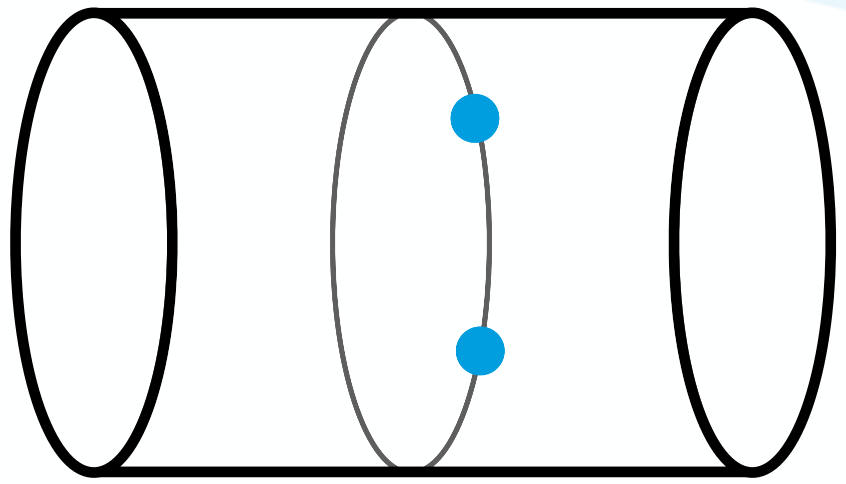
Input		**Output**
Δ	$\longrightarrow$	$a_{\mathcal{O}}$

2. Bootstrapping thermal CFTs

LIMIT $\mathbf{x = 0}$



2. Bootstrapping thermal CFTs

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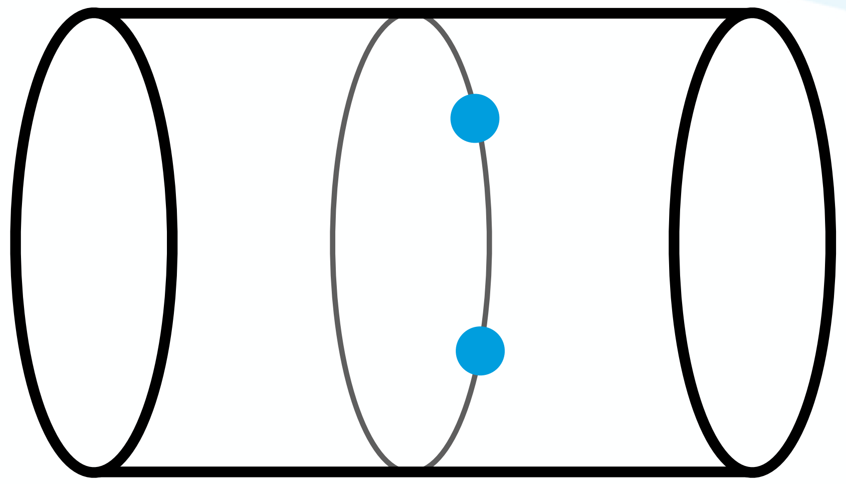
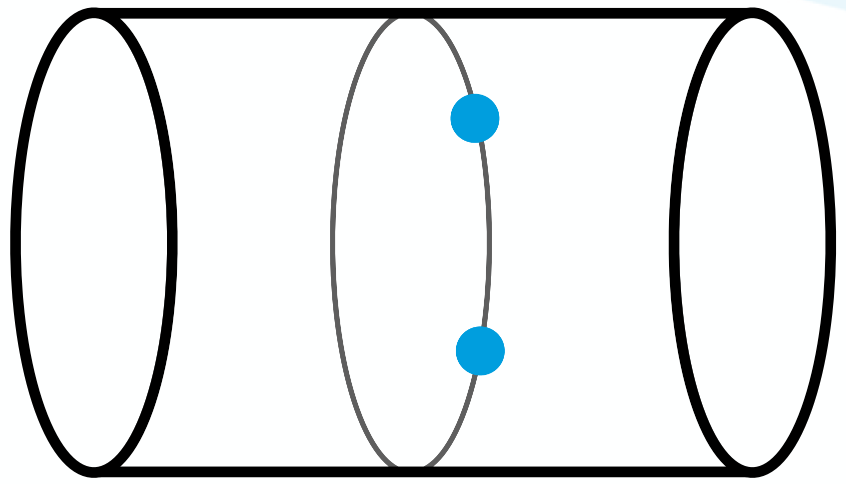


Goal: Derive a formula that encodes the **analytic structure**, while being **manifestly periodic** and containing the **thermal OPE coefficients**

2. Bootstrapping thermal CFTs

LIMIT $\mathrm{x} = 0$



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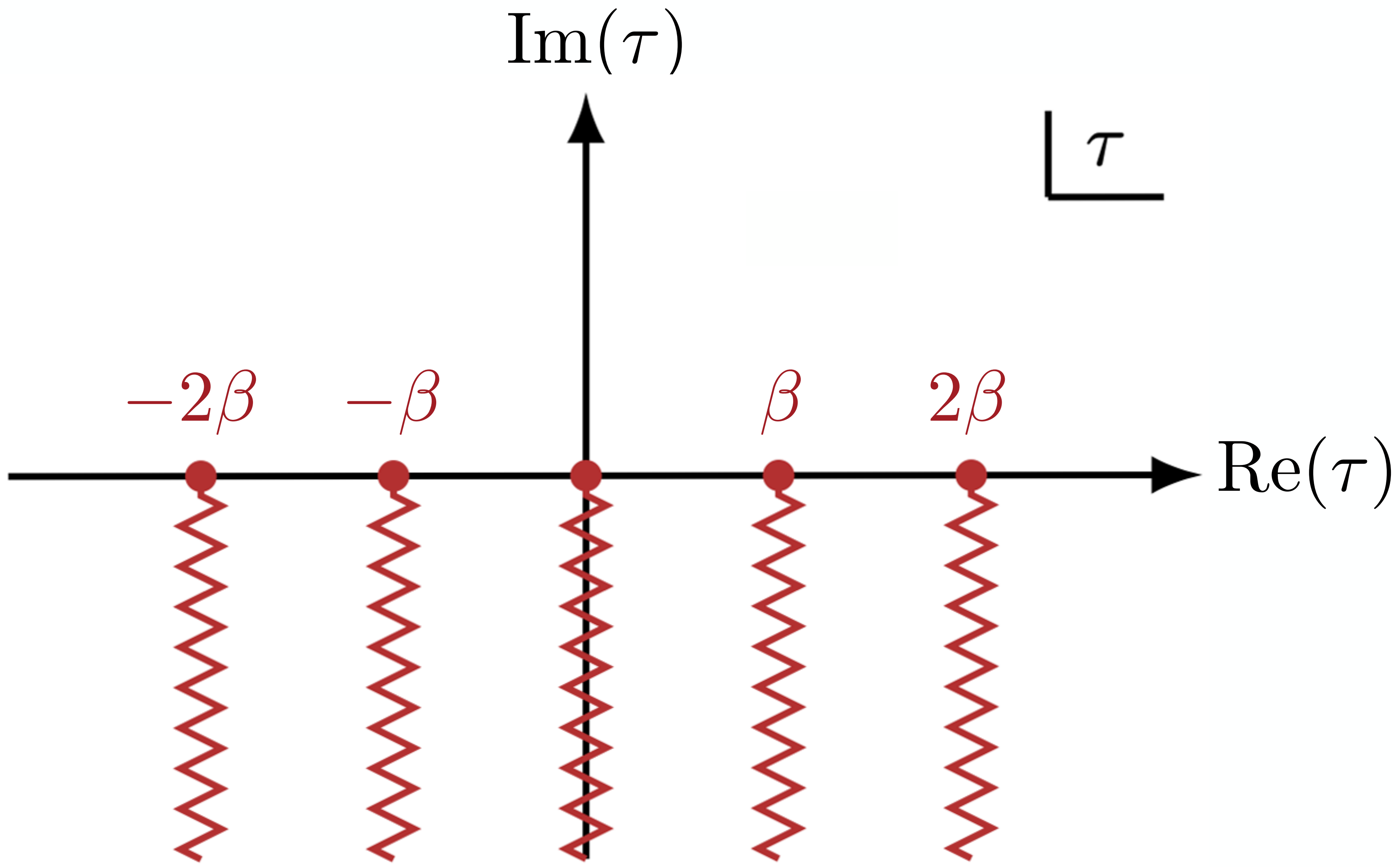

2. Bootstrapping thermal CFTs

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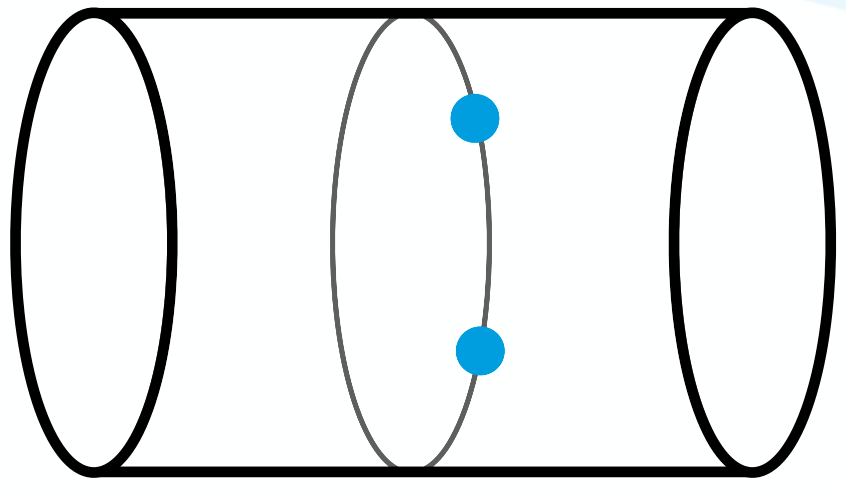

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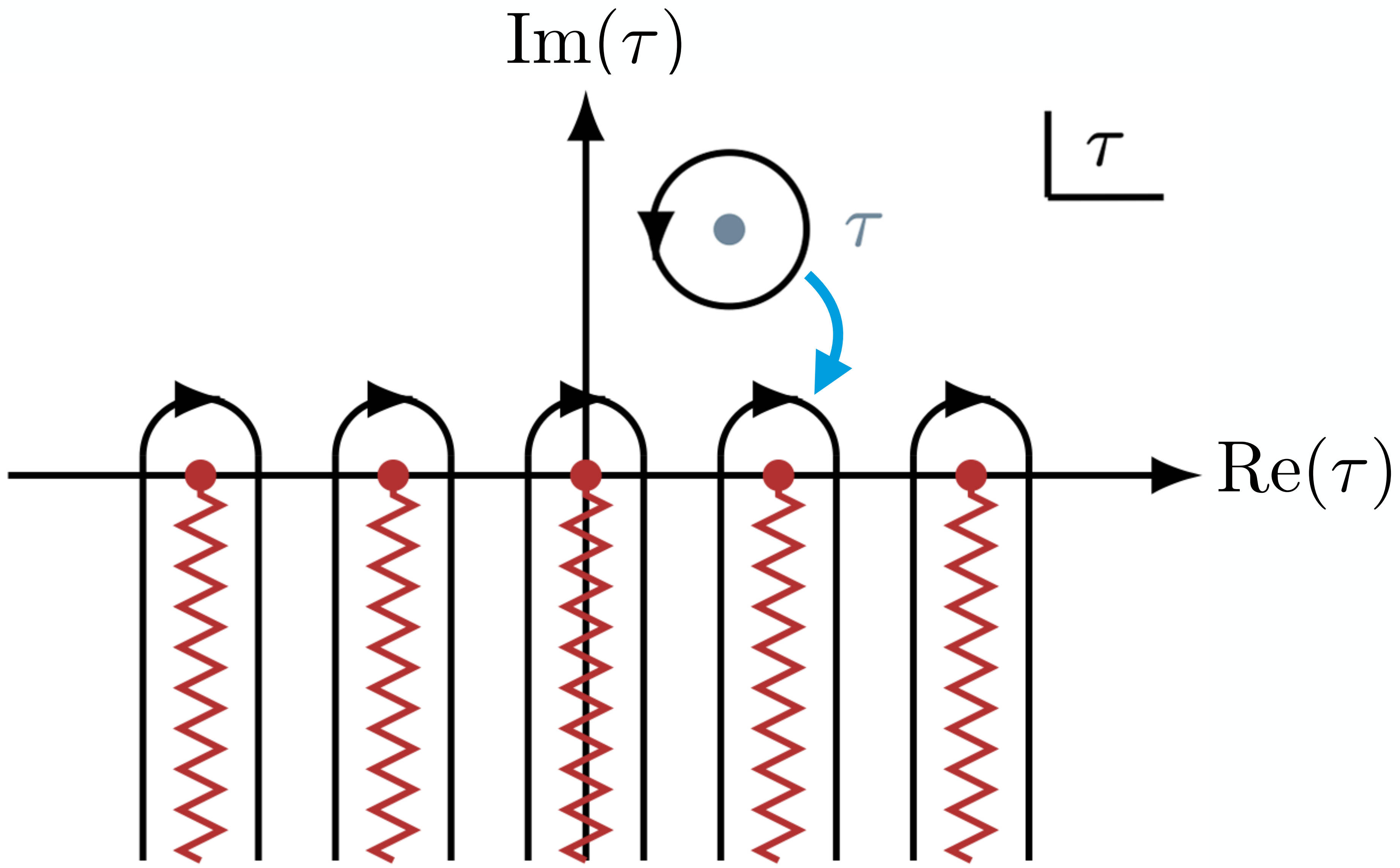

Dispersion relation:

$$g(\tau) = \sum_{m=-\infty}^{\infty} \int_{-i\infty}^{0} \frac{d\tau'}{2\pi i} \frac{\text{Disc } g(\tau')}{\tau' + m - \tau} + g_{\text{arcs}}(\tau)$$

with $\text{Disc}\, g(\tau) = \lim_{\varepsilon \to 0} \big(g(\tau + i\varepsilon) - g(\tau - i\varepsilon) \big)$

2. Bootstrapping thermal CFTs

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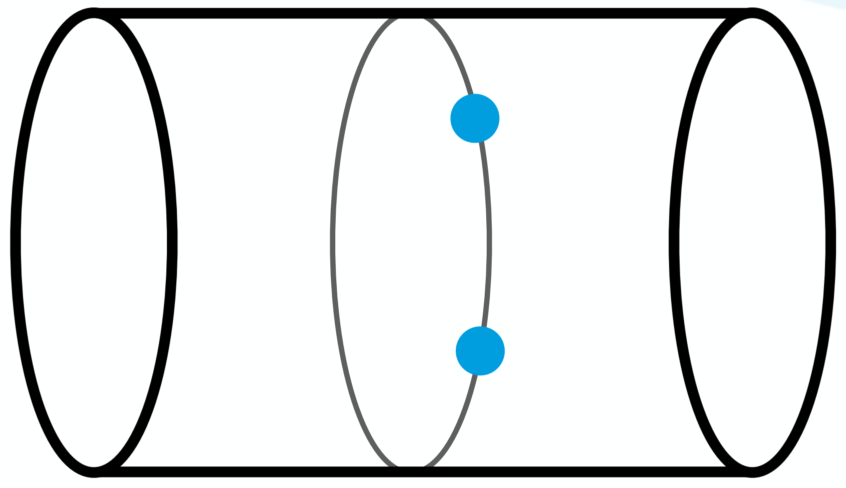

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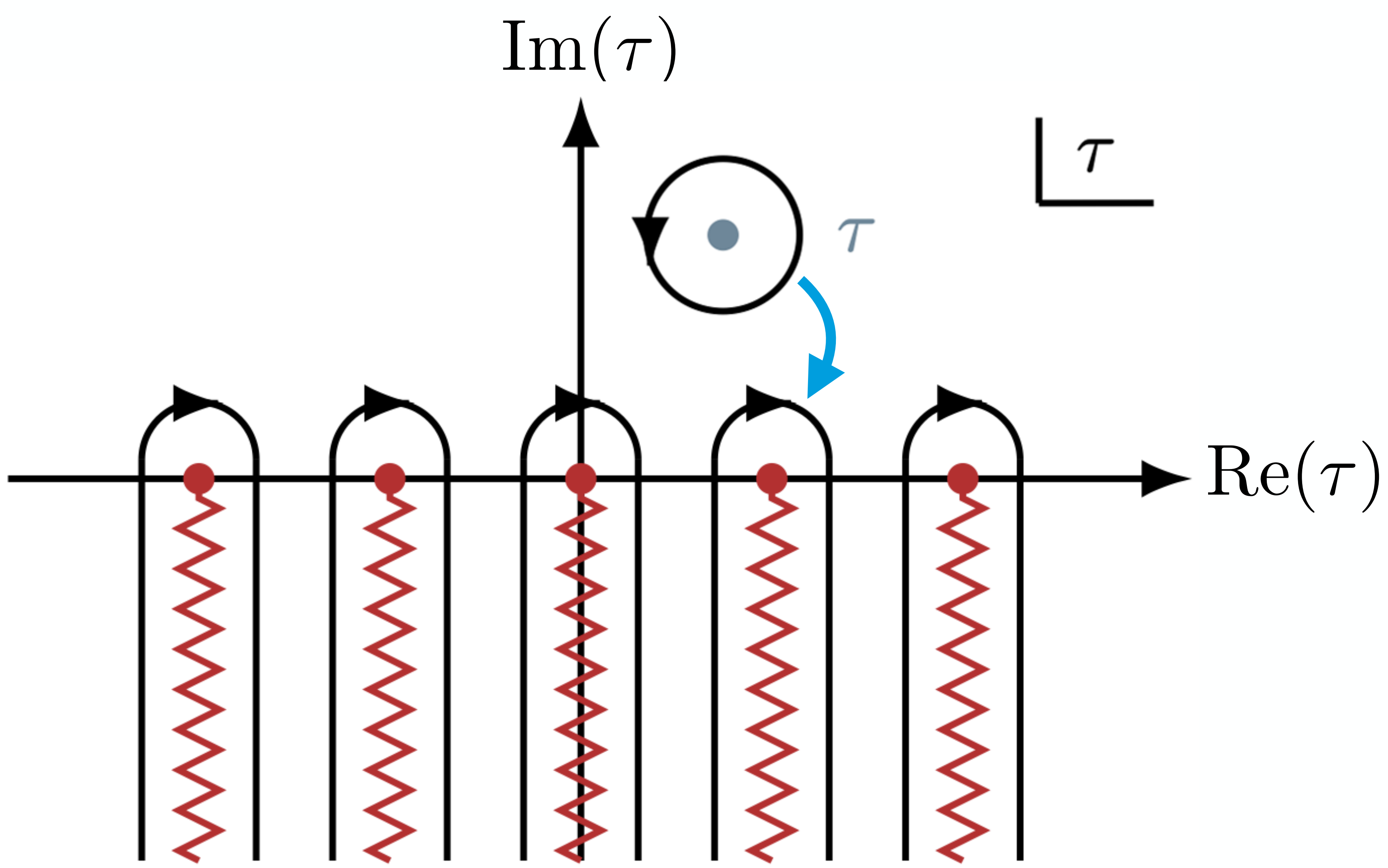

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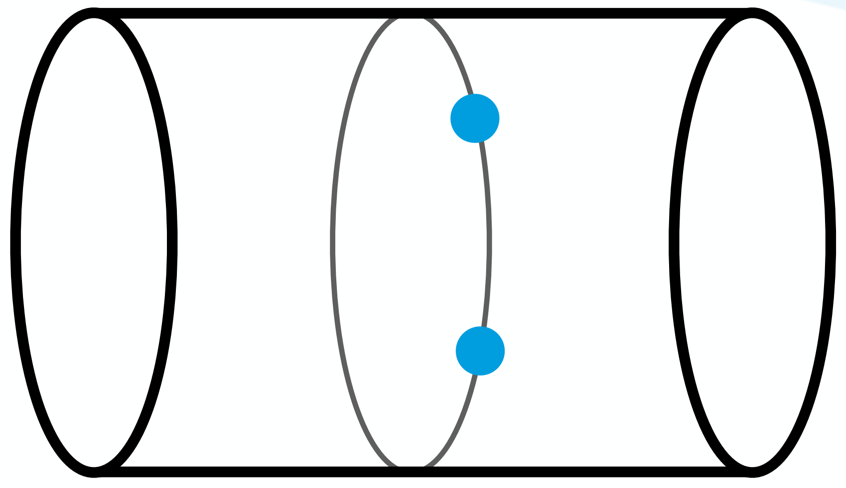
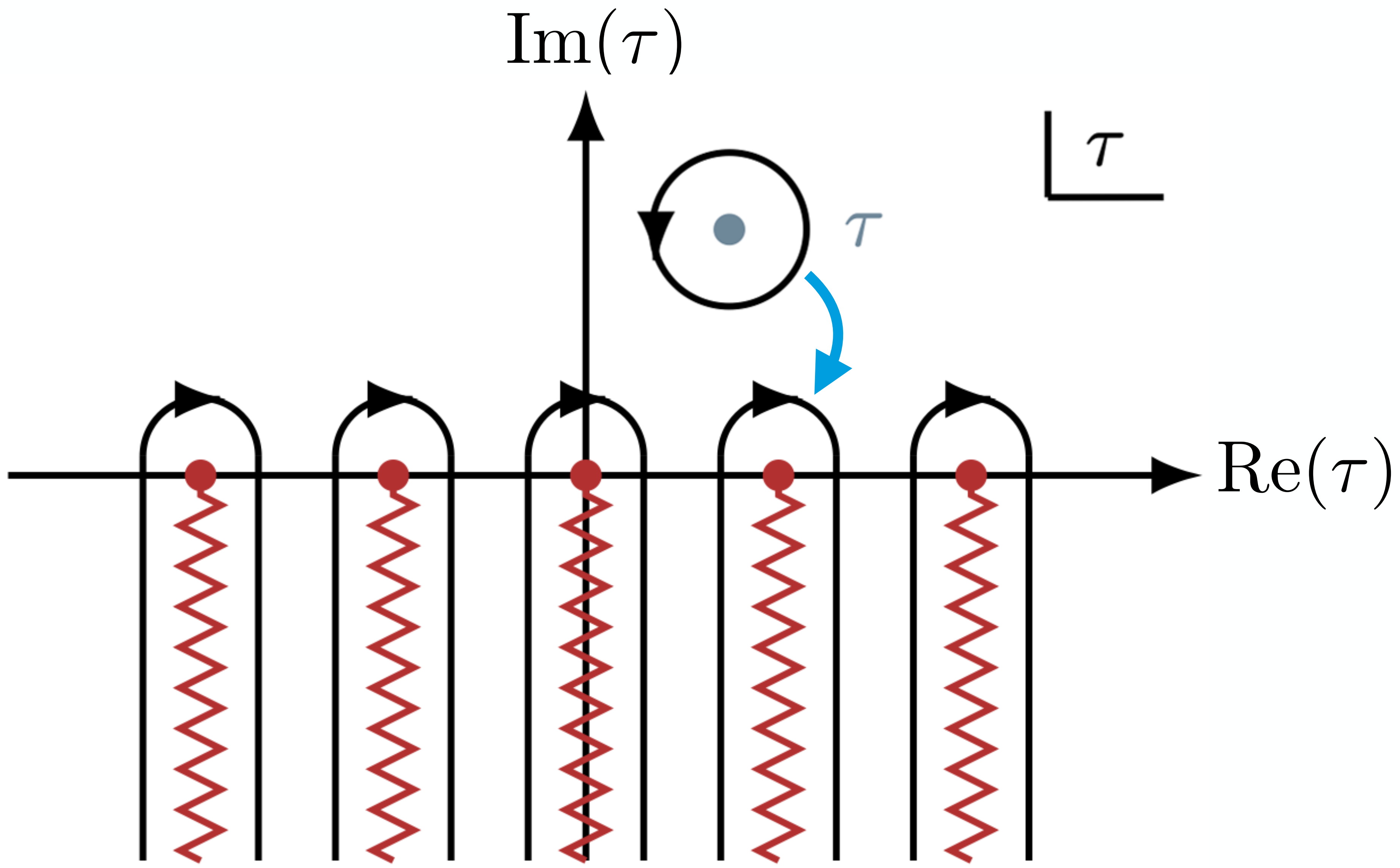


Analytic structure:



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Simpler object

2. Bootstrapping thermal CFTs

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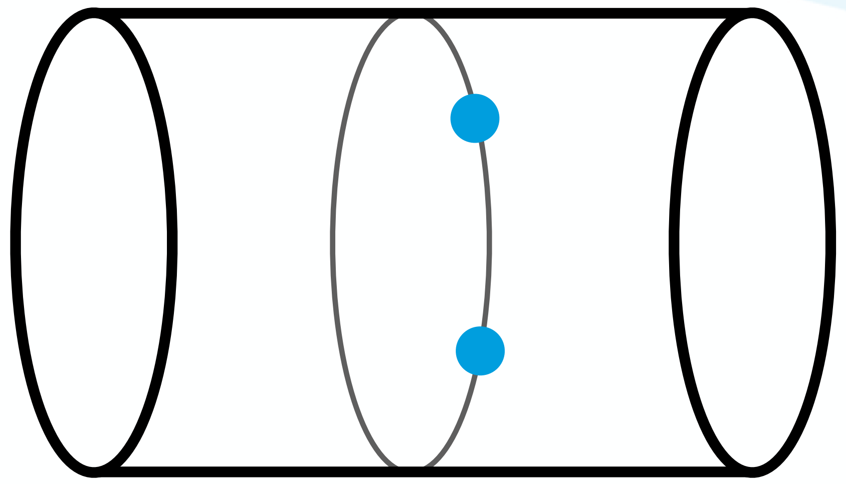

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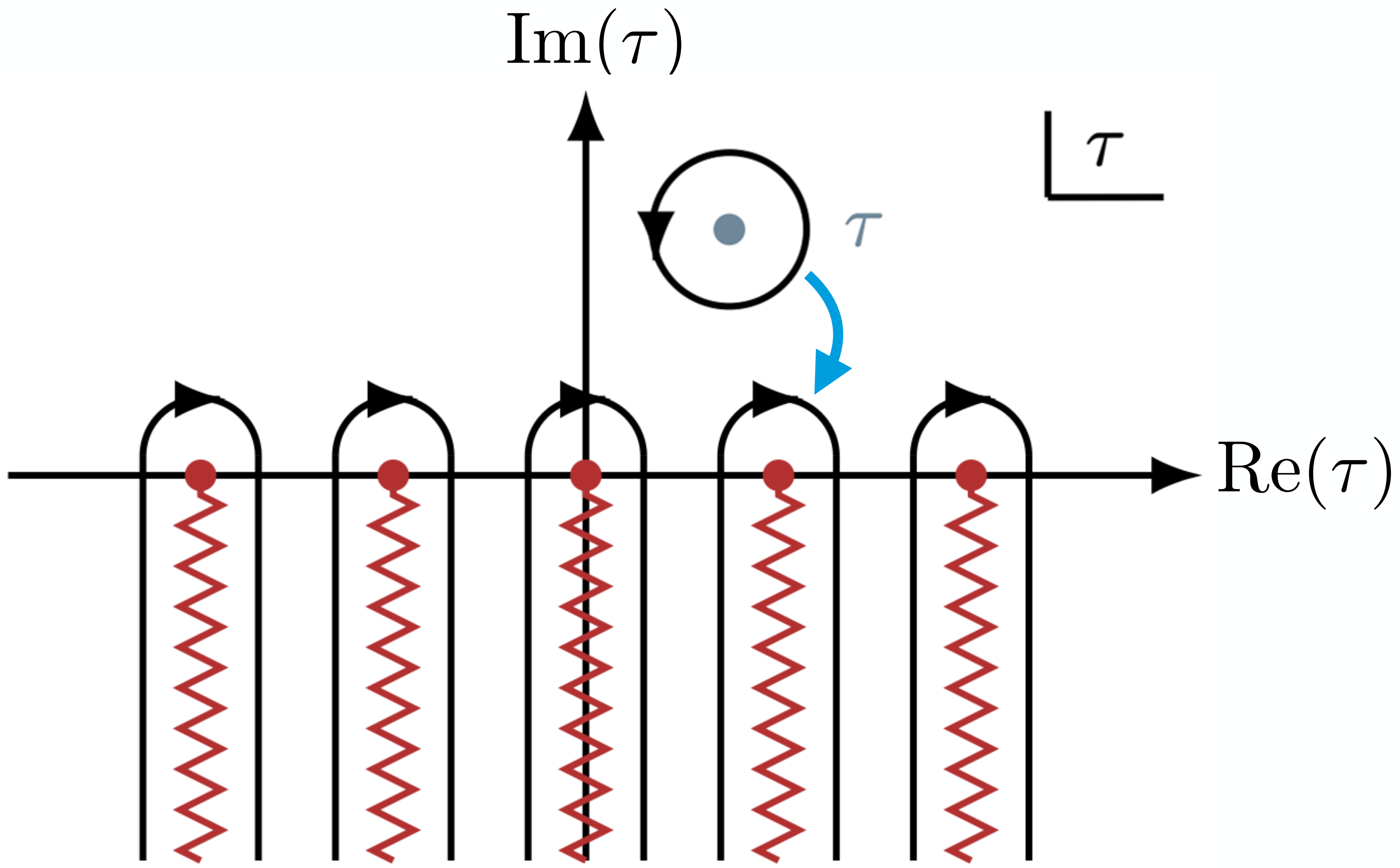

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Periodic

2. Bootstrapping thermal CFTs

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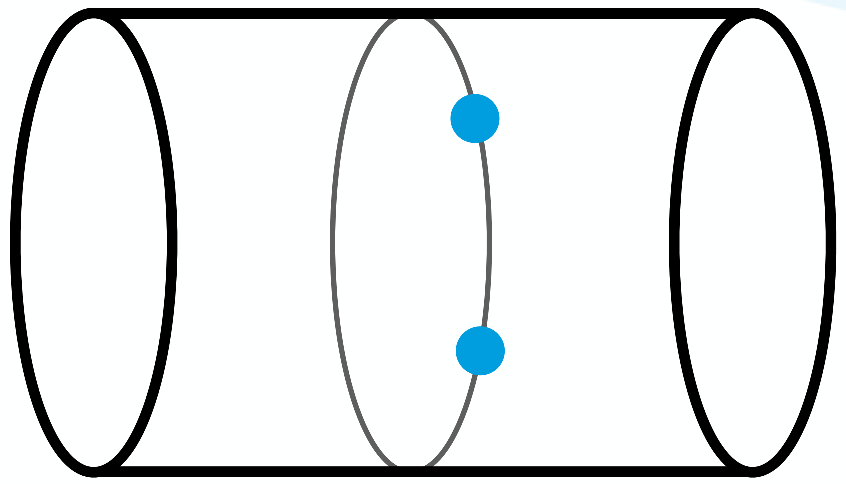

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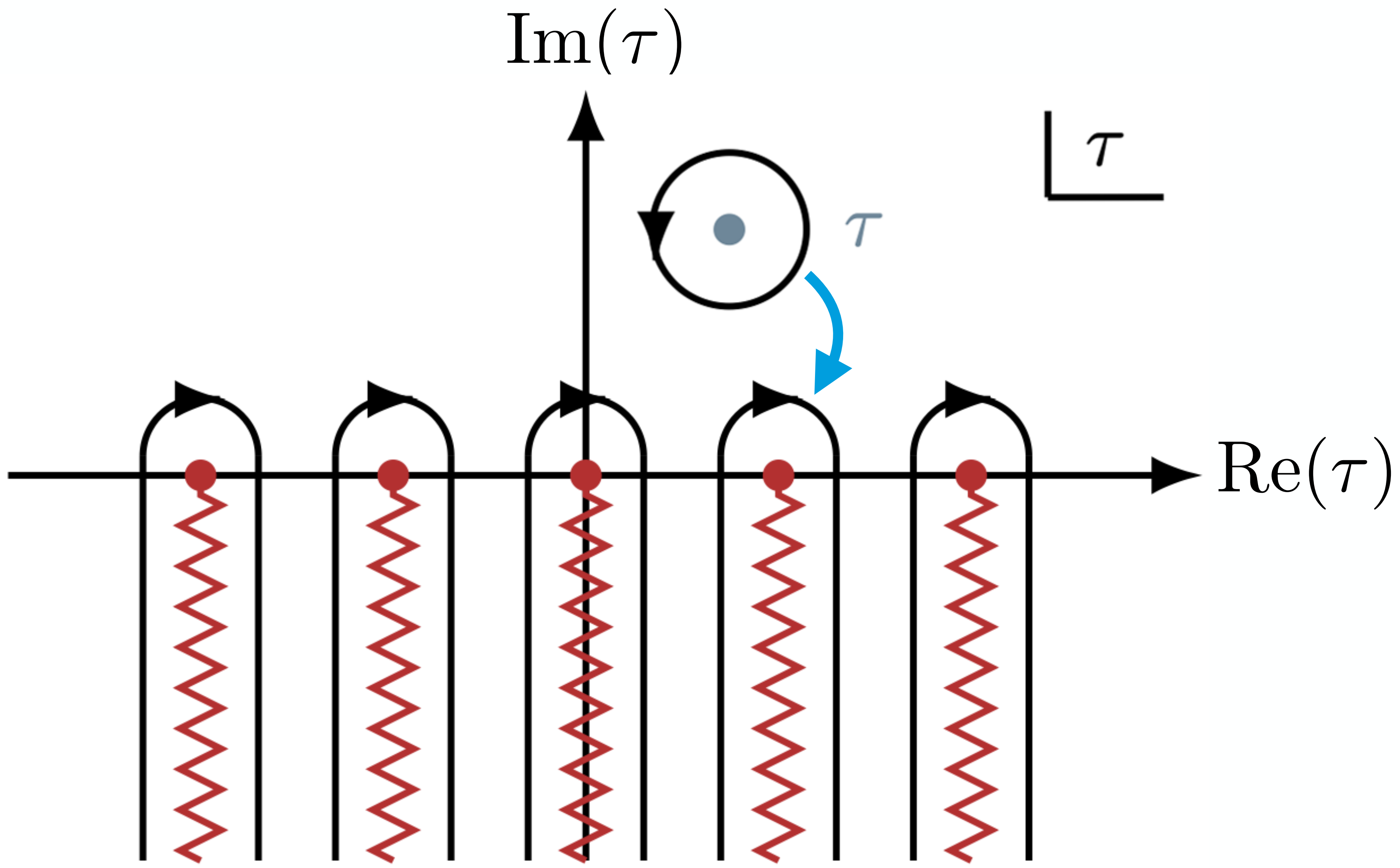

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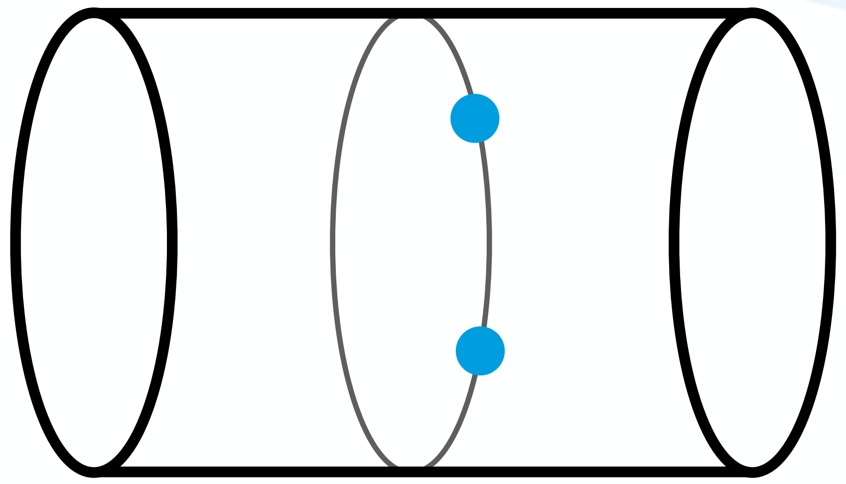
One can show that $g_{\mathrm{arcs}}(\tau) = \kappa = \mathrm{const.}$

2. Bootstrapping thermal CFTs

LIMIT $\mathbf{x = 0}$: OPE



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2. Bootstrapping thermal CFTs

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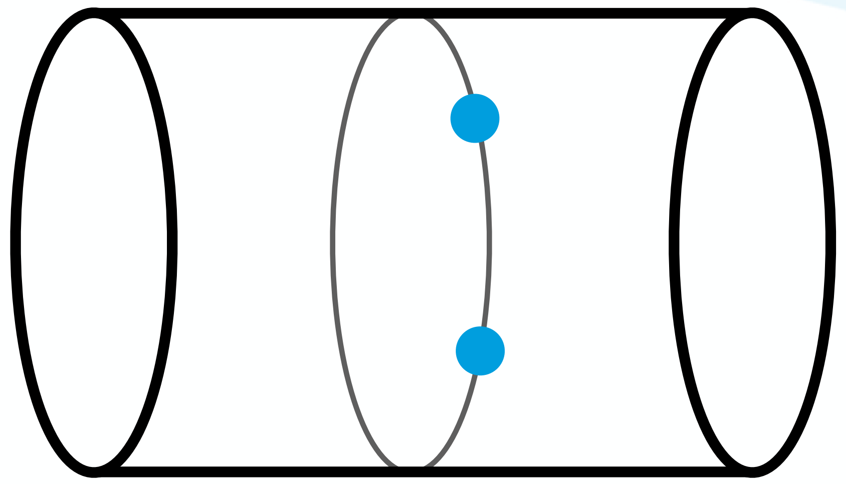

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Insert OPE

$$= \sum_{\Delta} a_\Delta \big(\zeta_H(2\Delta_\phi - \Delta, \tau) + \zeta_H(2\Delta_\phi - \Delta, 1 - \tau)\big) + \kappa$$

2. Bootstrapping thermal CFTs

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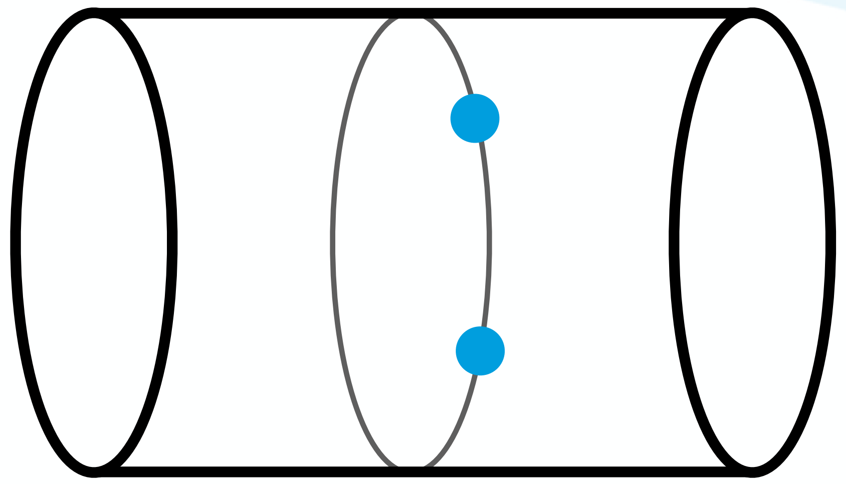

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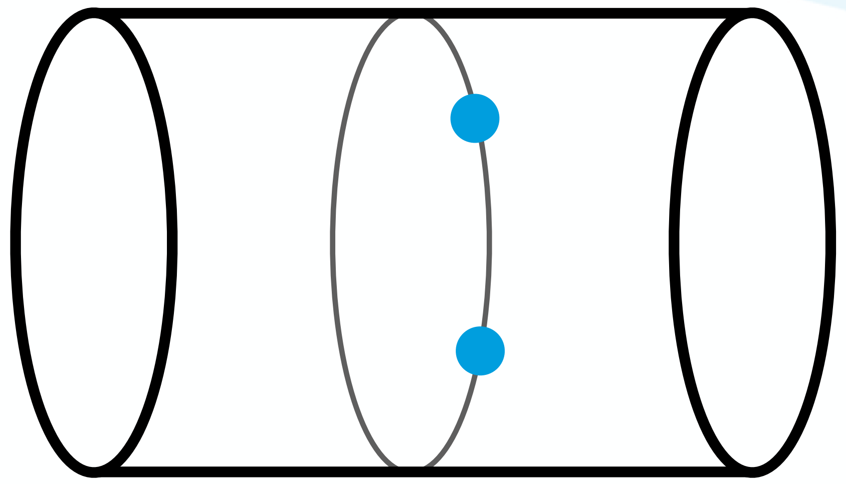

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Any thermal correlator can be expanded into GFF correlators

2. Bootstrapping thermal CFTs

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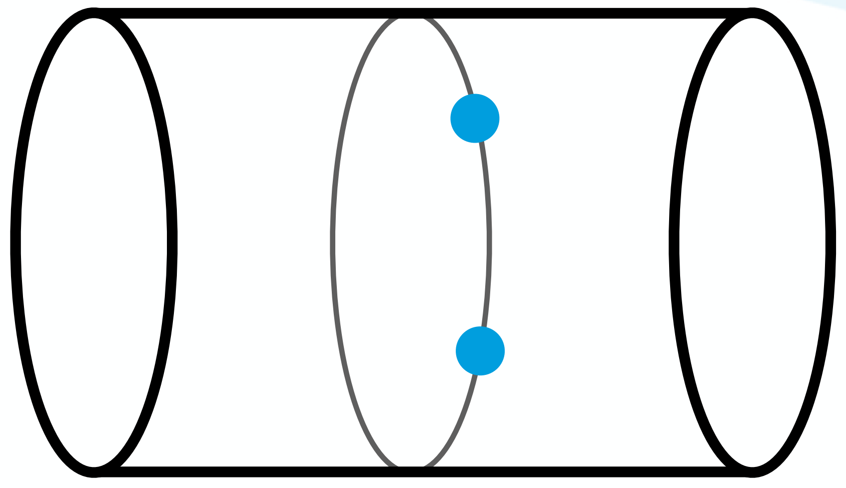

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- Analytic structure; ✓

2. Bootstrapping thermal CFTs

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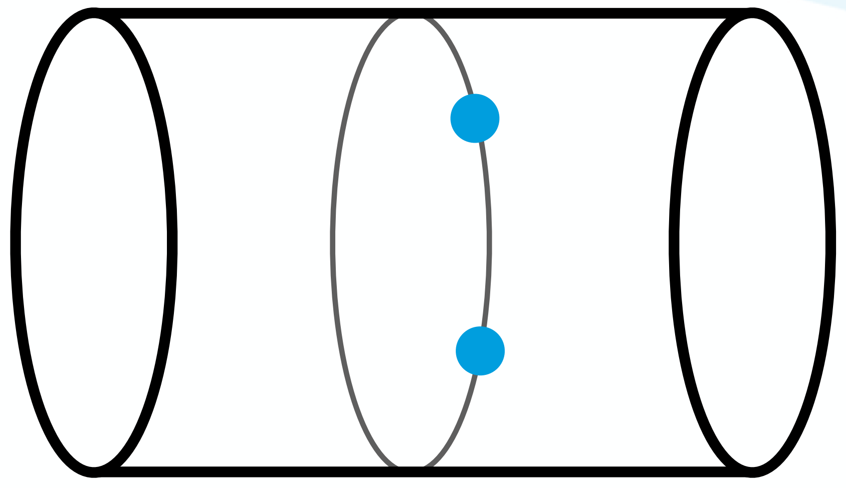

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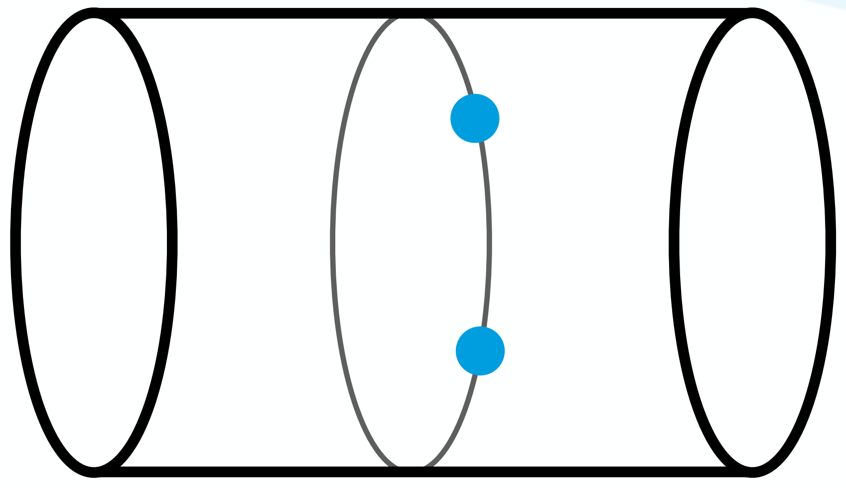
- Periodicity; ✓

2. Bootstrapping thermal CFTs

LIMIT $\mathrm{x} = 0$: OPE



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2. Bootstrapping thermal CFTs

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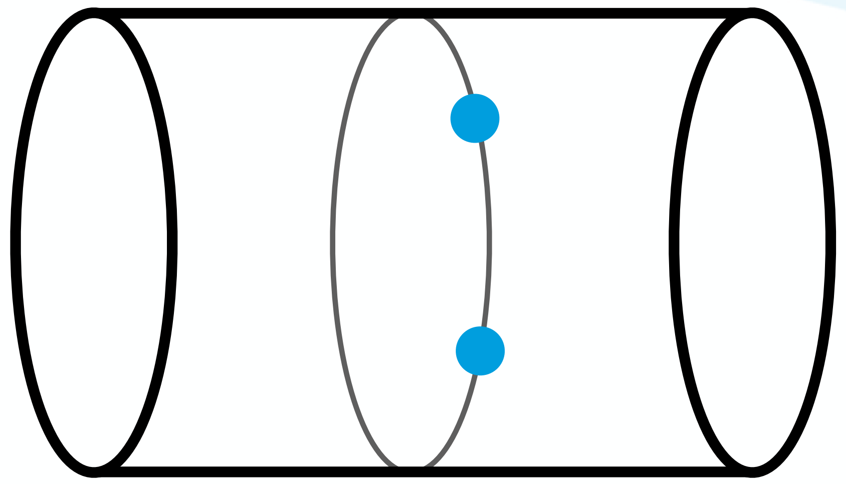

2. Bootstrapping thermal CFTs

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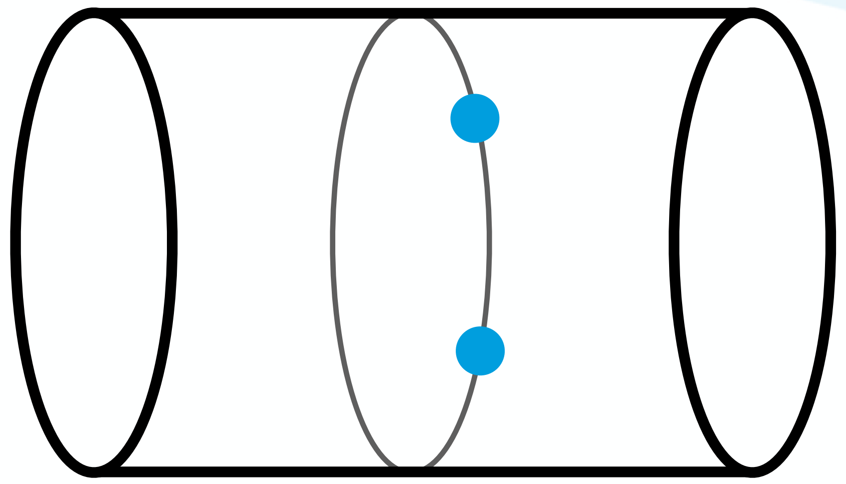

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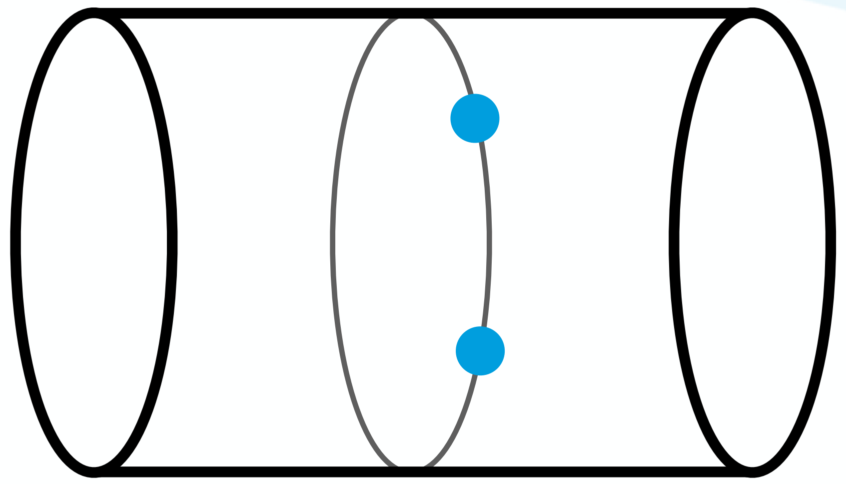
Spectrum: $\phi \times \phi \to \mathbb{1} + [\phi\phi]_{n,J}$ with $\Delta_{n,J} = 2\Delta_\phi + 2n + J$

2. Bootstrapping thermal CFTs

EXAMPLE: GFF



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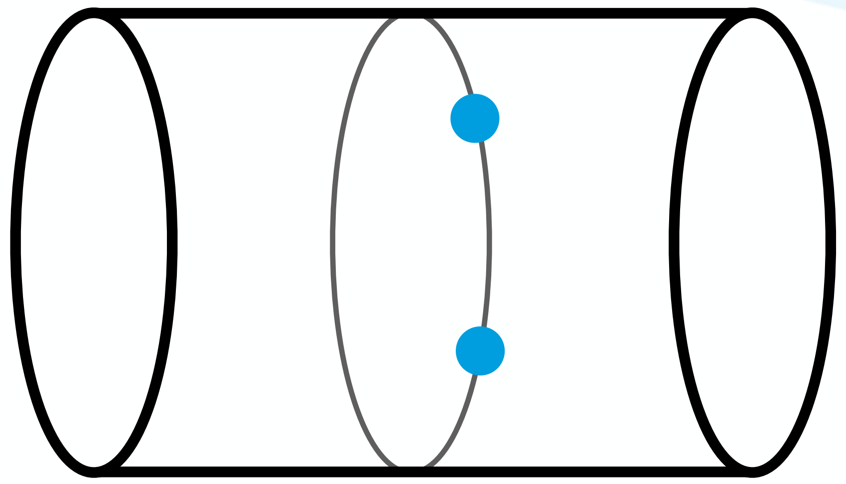

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2. Bootstrapping thermal CFTs

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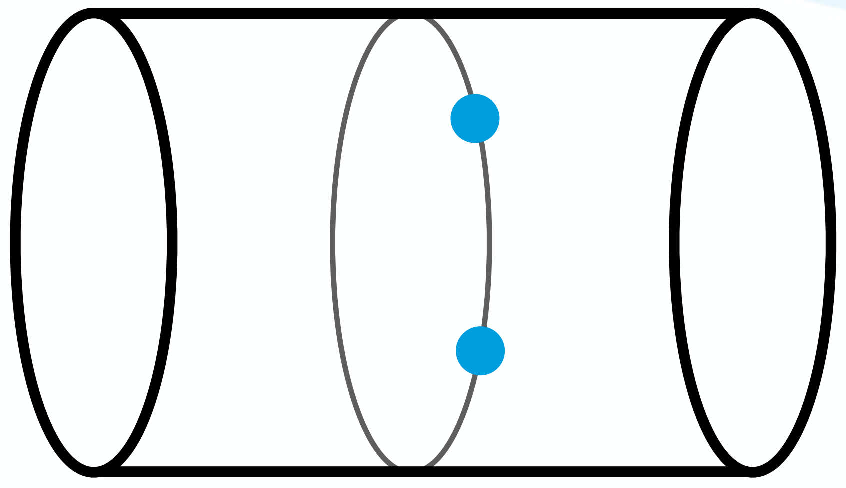

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Final result: $$g(\tau) = \zeta_H(2\Delta_\phi, \tau) + \zeta_H(2\Delta_\phi, 1-\tau) \qquad (\kappa = 0 \;\text{ in this case})$$

2. Bootstrapping thermal CFTs

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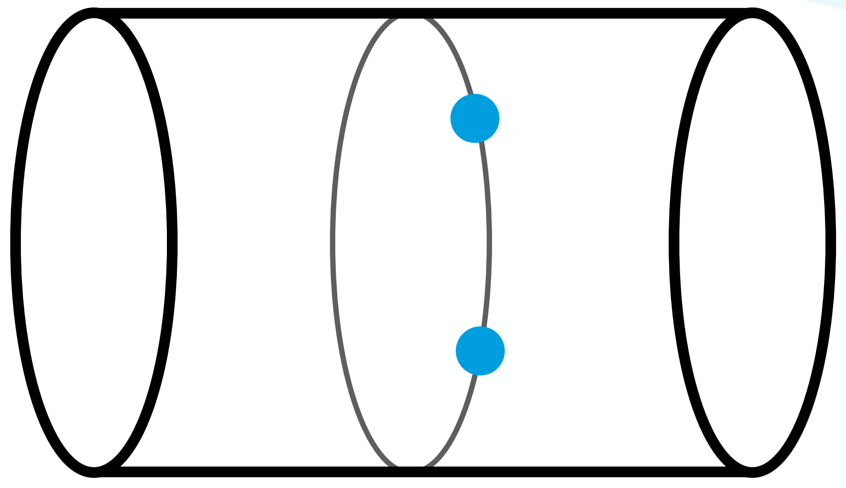

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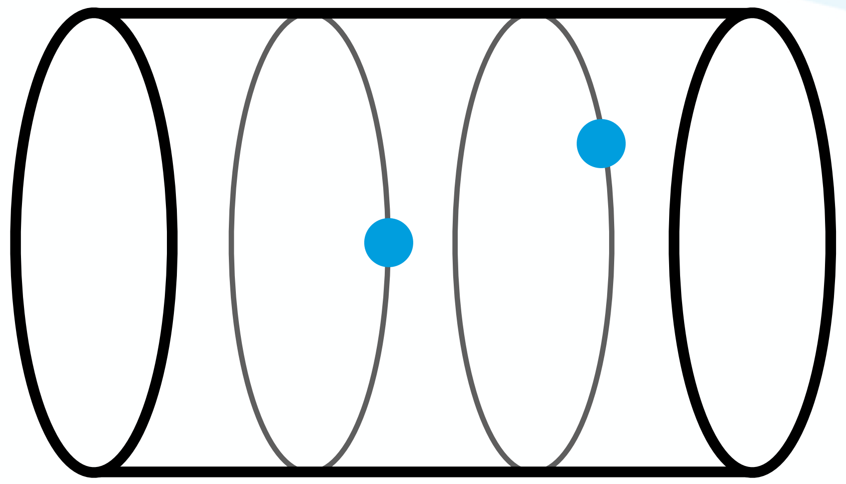
Lesson: $\mathbb{1} \xrightarrow{\text{DR}} [\phi\phi]_{n,J}$

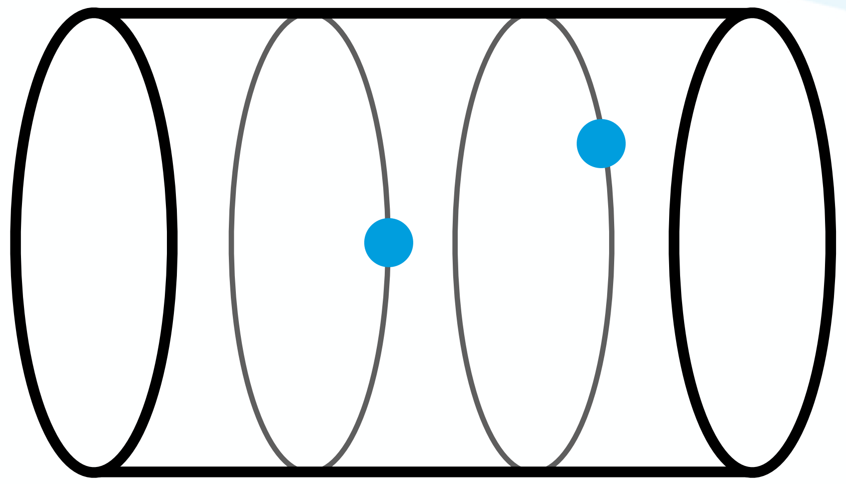
The double-twist operators are reconstructed from the identity

GENERAL CASE $\mathbf{x} \neq \mathbf{0}$



2. Bootstrapping thermal CFTs

GENERAL CASE $\mathbf{x} \neq \mathbf{0}$



Dispersion relation:

$$g(r,w) = \sum_{m=-\infty}^{\infty} \int_0^r \frac{dw'}{2\pi i} K(r,w,w',m)\, \mathrm{Disc}\, g(r,w') + \kappa$$

(variables: $r^2 = z\bar{z}\,, w^2 = \frac{z}{\bar{z}}$, with $z = \tau + ix\,, \bar{z} = \tau - ix$)

2. Bootstrapping thermal CFTs

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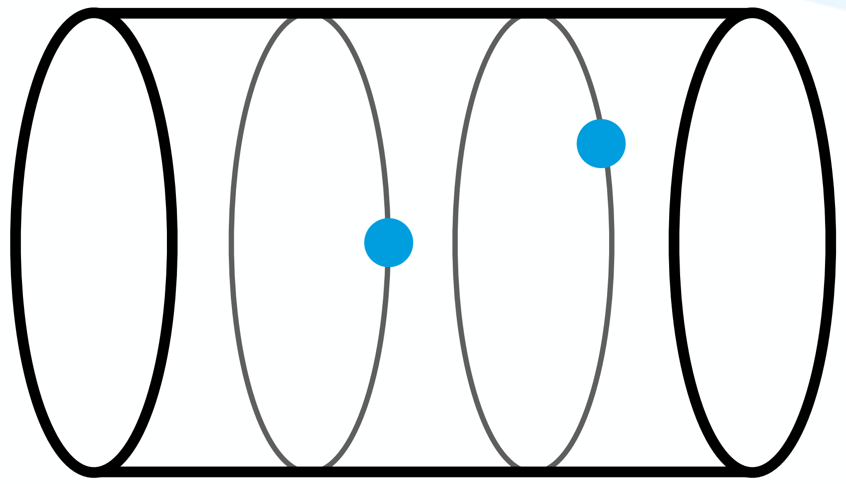

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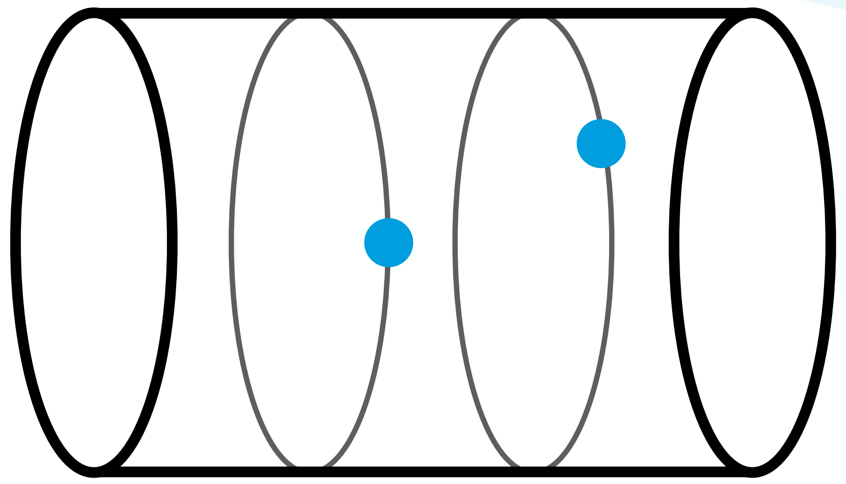
Known kernel

2. Bootstrapping thermal CFTs

GENERAL CASE $\mathbf{x \neq 0}$



Dispersion relation:

$$g(r,w) = \boxed{\sum_{m=-\infty}^{\infty}} \int_0^r \frac{dw'}{2\pi i} K(r,w,w',m)\,\mathrm{Disc}\, g(r,w') + \kappa$$

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Periodic

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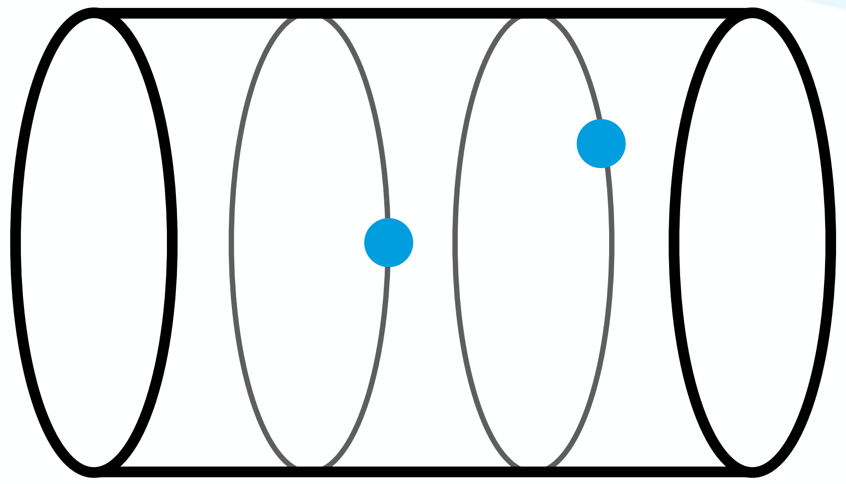

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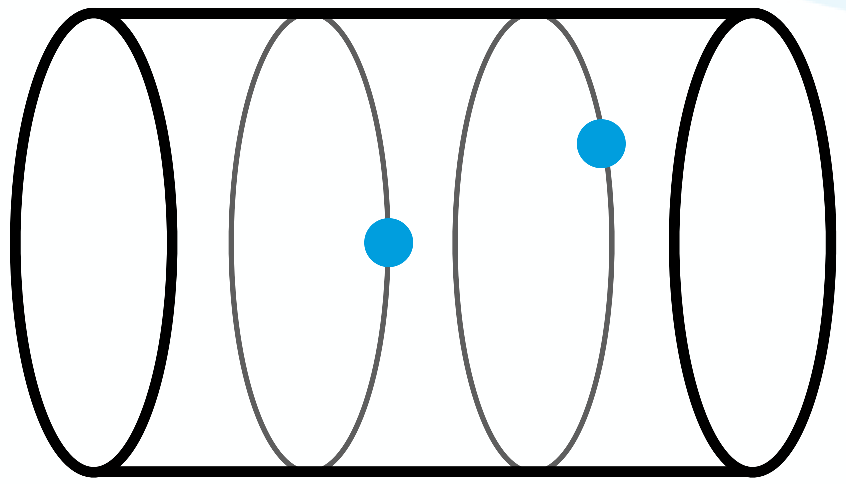

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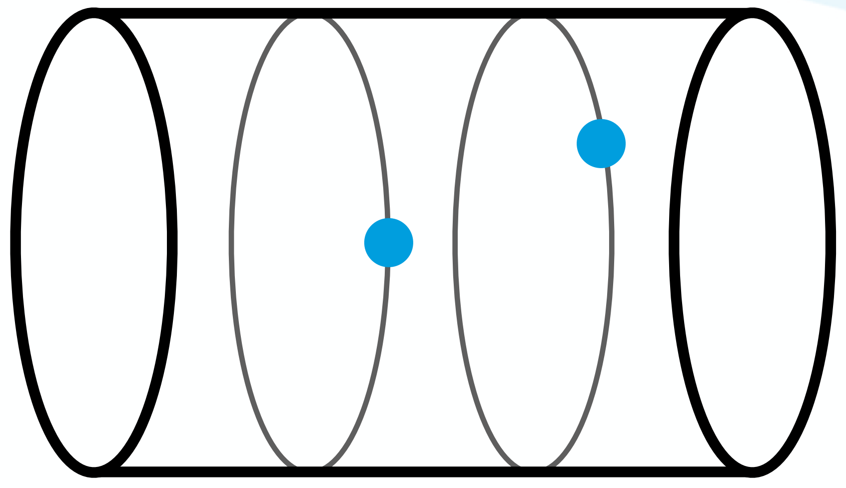

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- Analytic structure; ✓
- Periodicity; ✓
- OPE. ✓

3. Applications

3. Applications

O(N) MODEL IN $\mathbf{d} = 4 - \varepsilon$

3. Applications

O(N) MODEL IN $\mathrm{d} = 4 - \varepsilon$

Review:
- Action: $$S = \int d^dx \left(\frac{1}{2} \partial_\mu \phi^I \partial_\mu \phi^I + \frac{\lambda}{4!} \phi^I \phi^I \phi^J \phi^J \right) \qquad (I, J = 1, \ldots, N)$$
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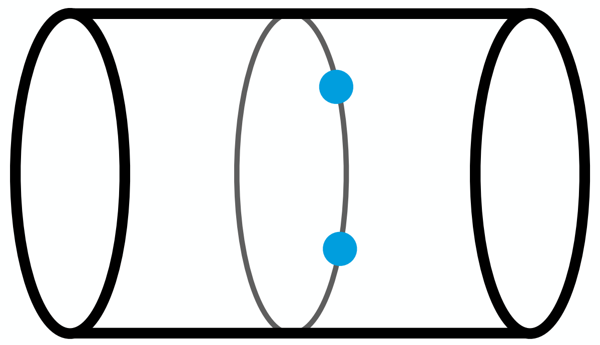

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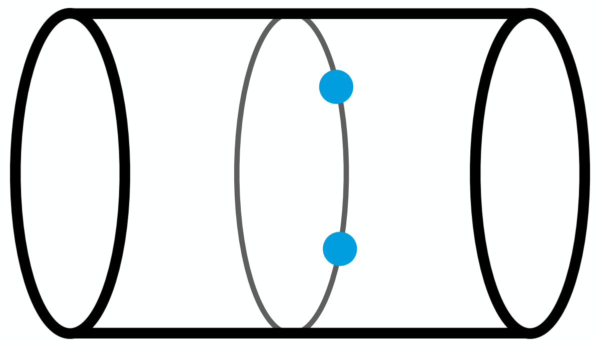

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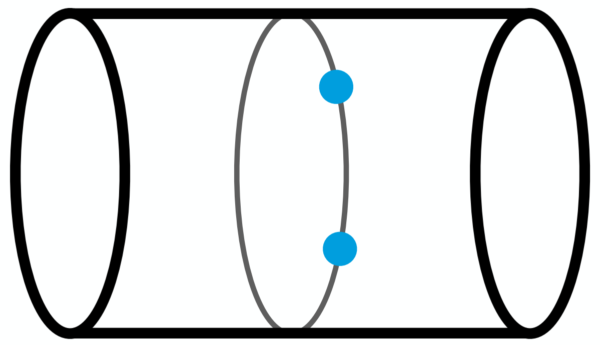
free theory



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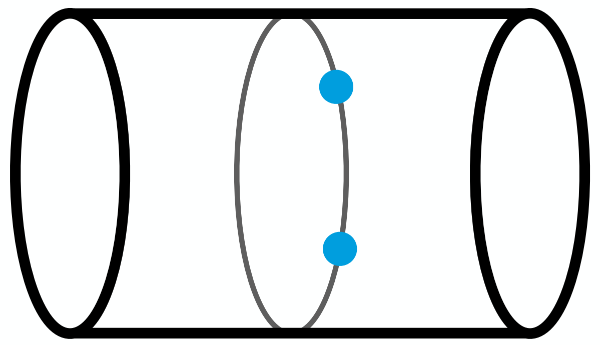
free theory **only** $J = 0$ (ϕ^2) **receives anomalous dimensions**



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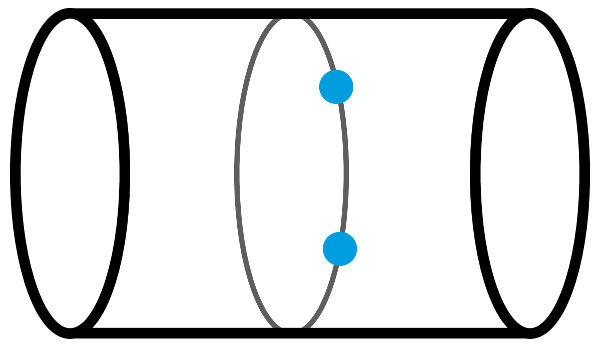

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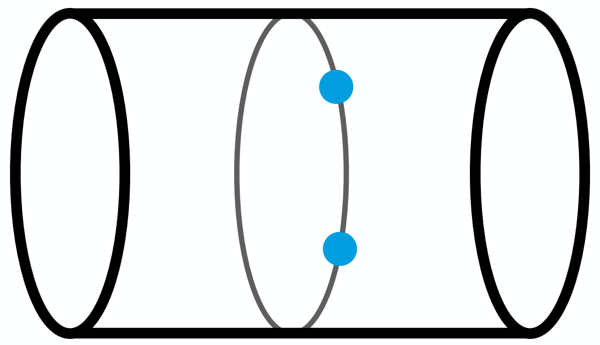

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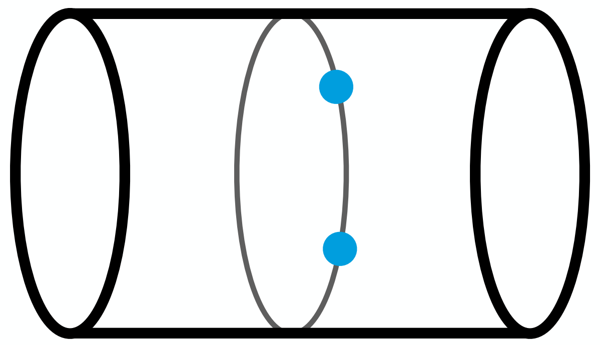

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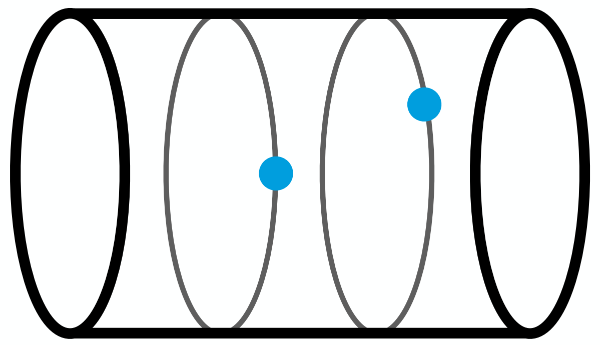

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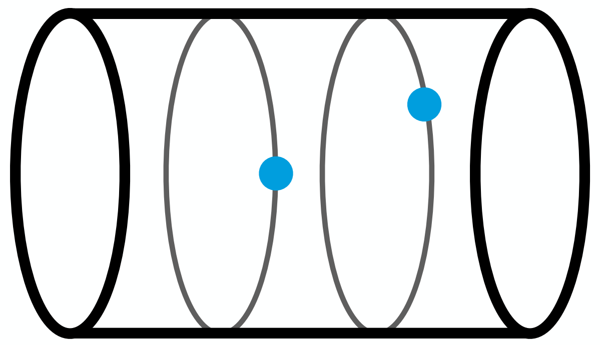

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- Matches Feynman diagrams

3. Applications

4*d* HOLOGRAPHIC THEORIES

[JB, Bozkurt, Marchetto, Miscioscia, Pomoni, WIP]

3. Applications

4*d* HOLOGRAPHIC THEORIES

[JB, Bozkurt, Marchetto, Miscioscia, Pomoni, WIP]

A long standing problem: $\langle\, \phi(0,0)\phi(\tau,x)\,\rangle_\beta$ at $\lambda = \infty$ for Δ_ϕ integer $(\mathcal{N}=4$ SYM$)$

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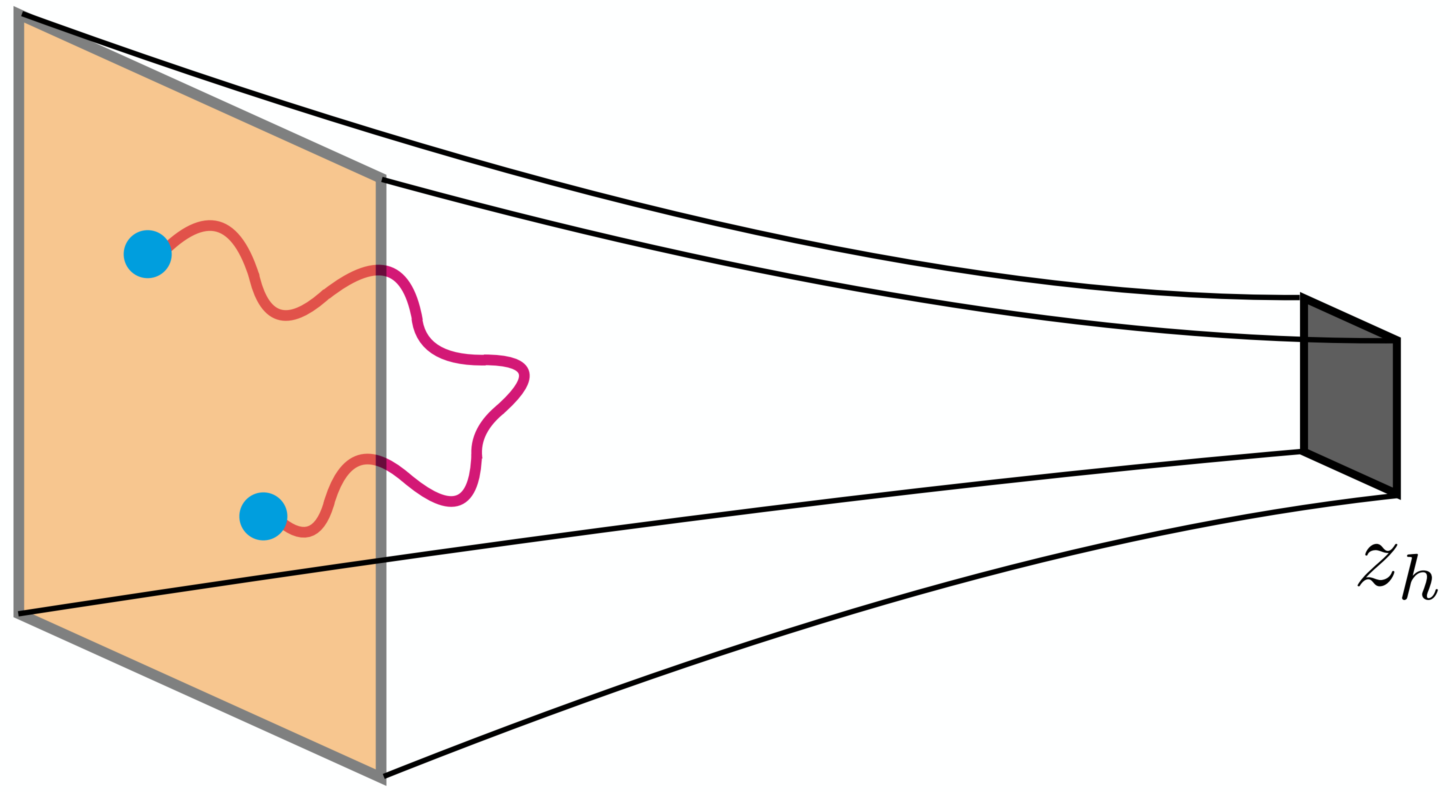

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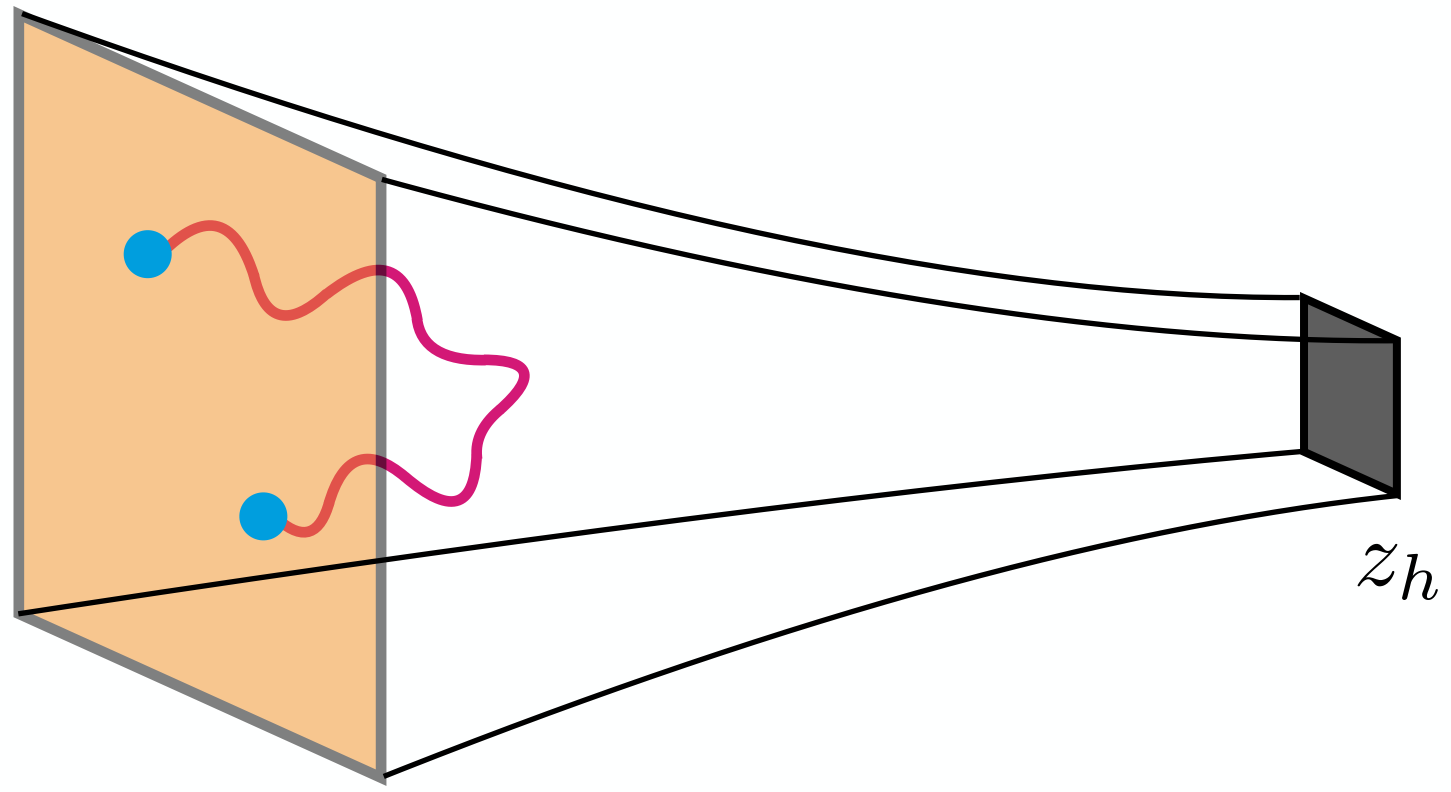

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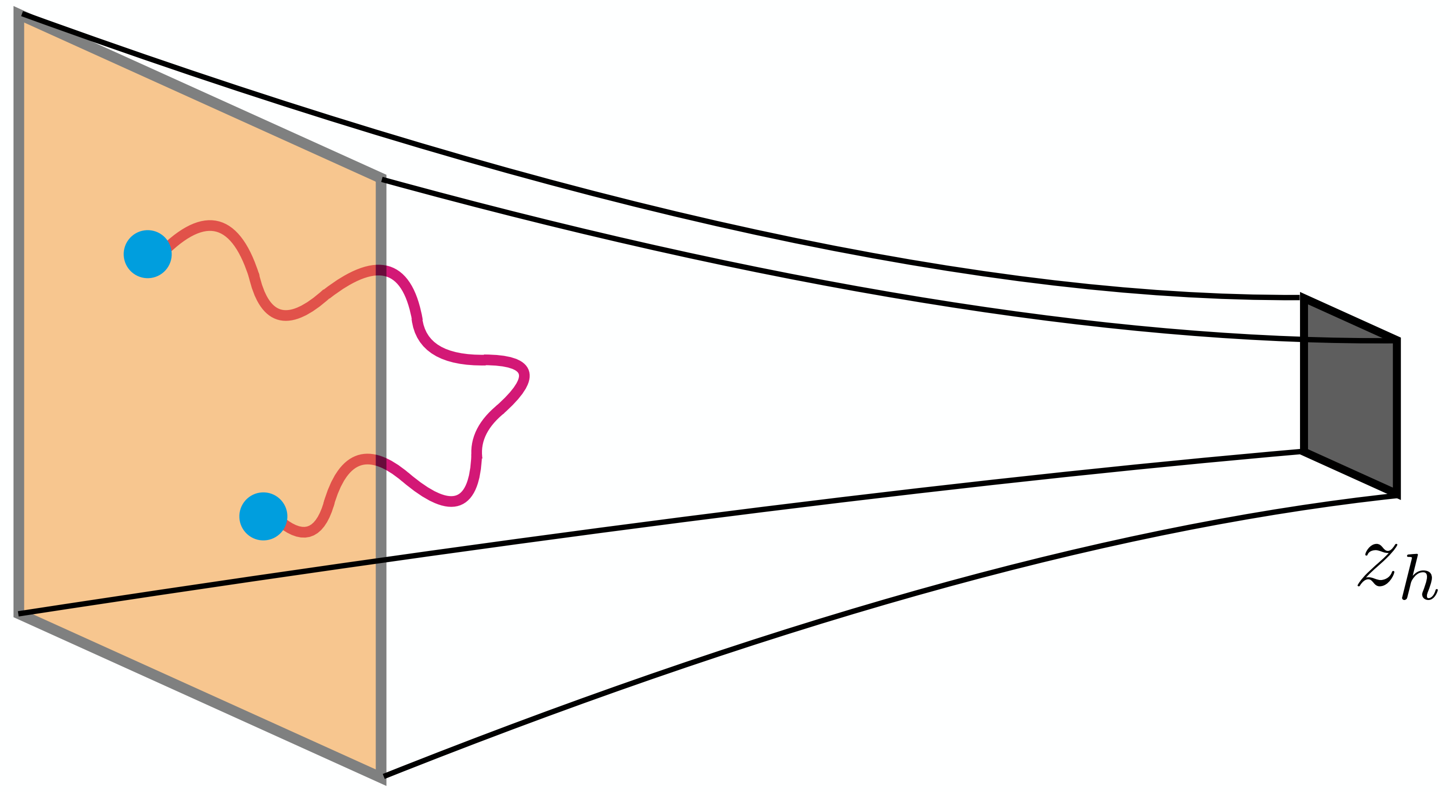

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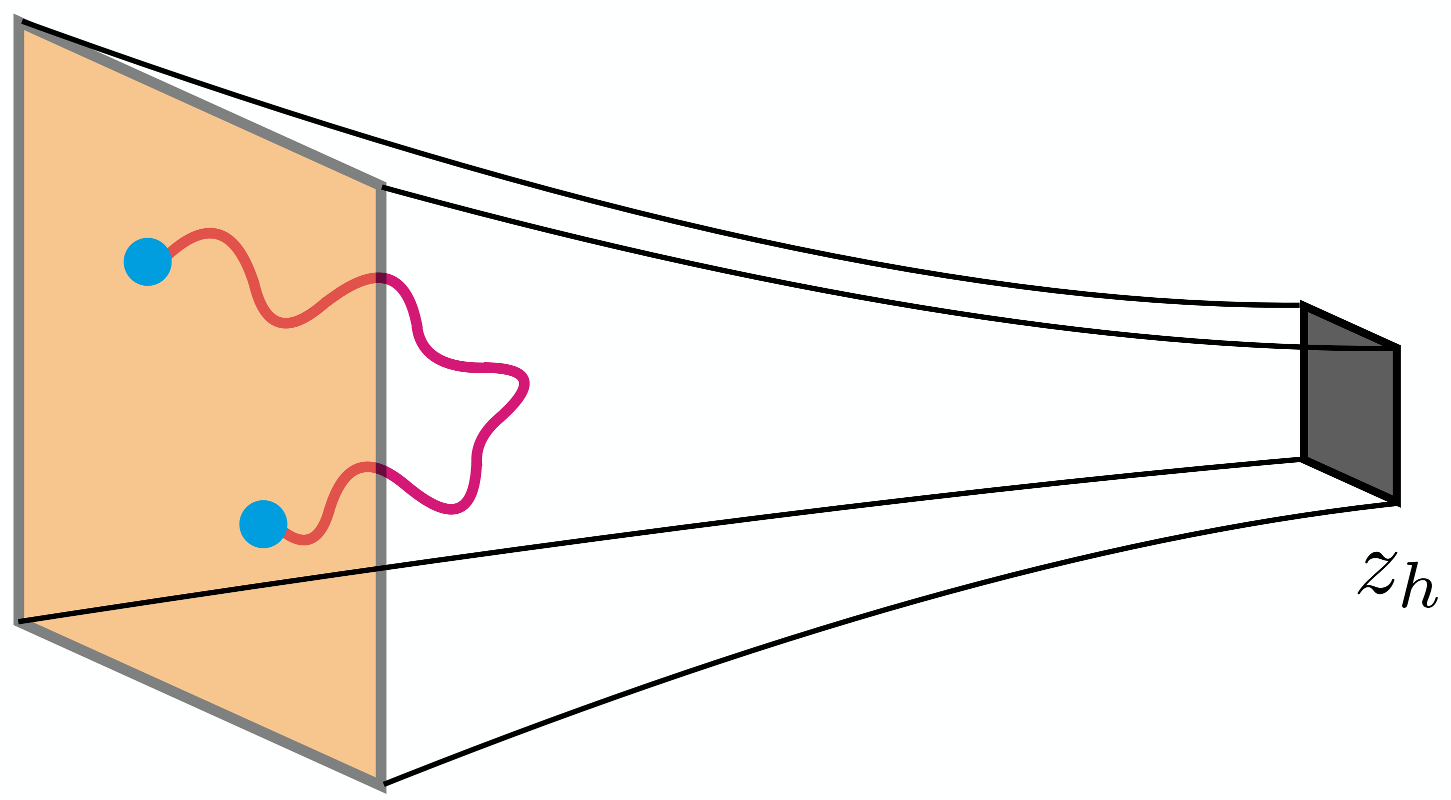

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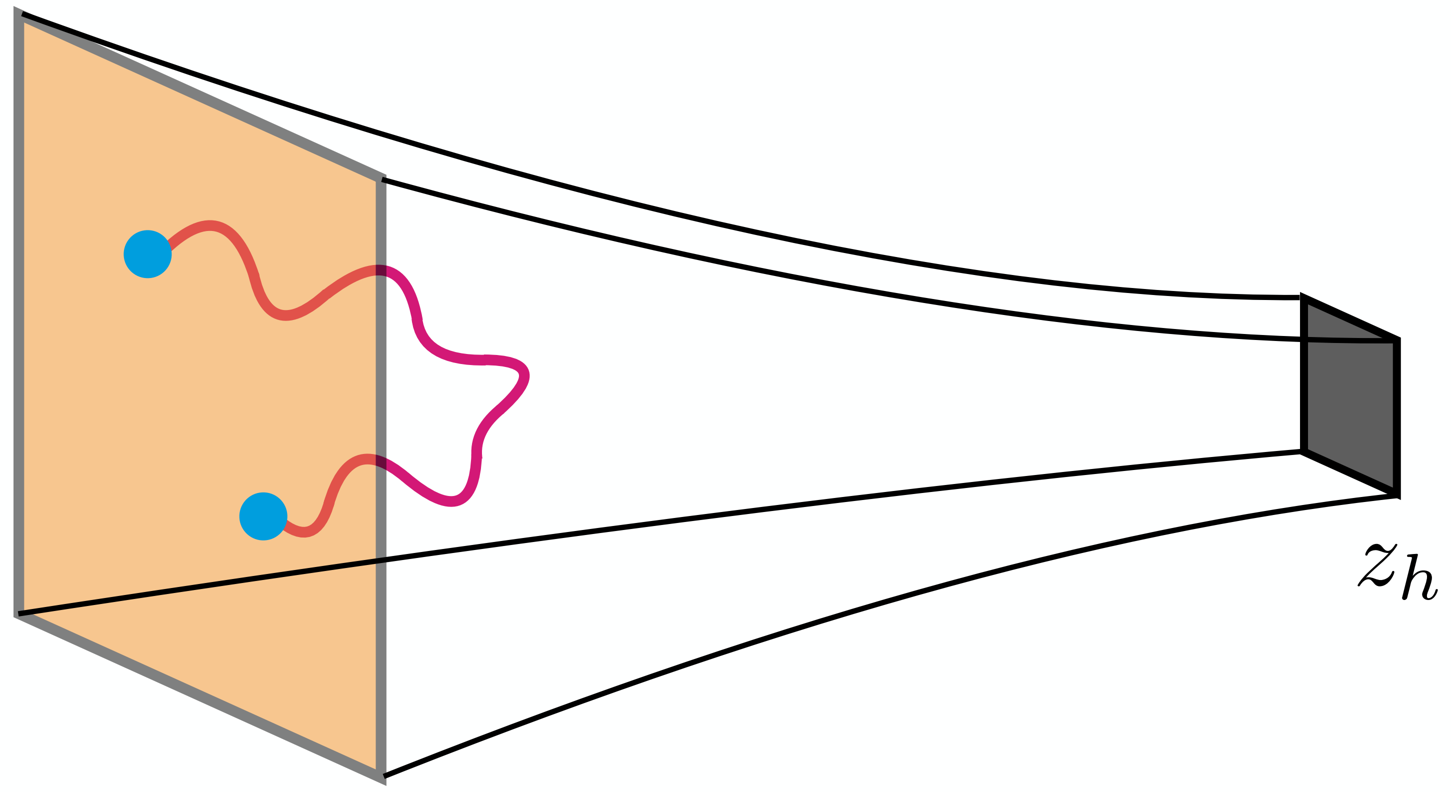

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$\downarrow$

double-trace

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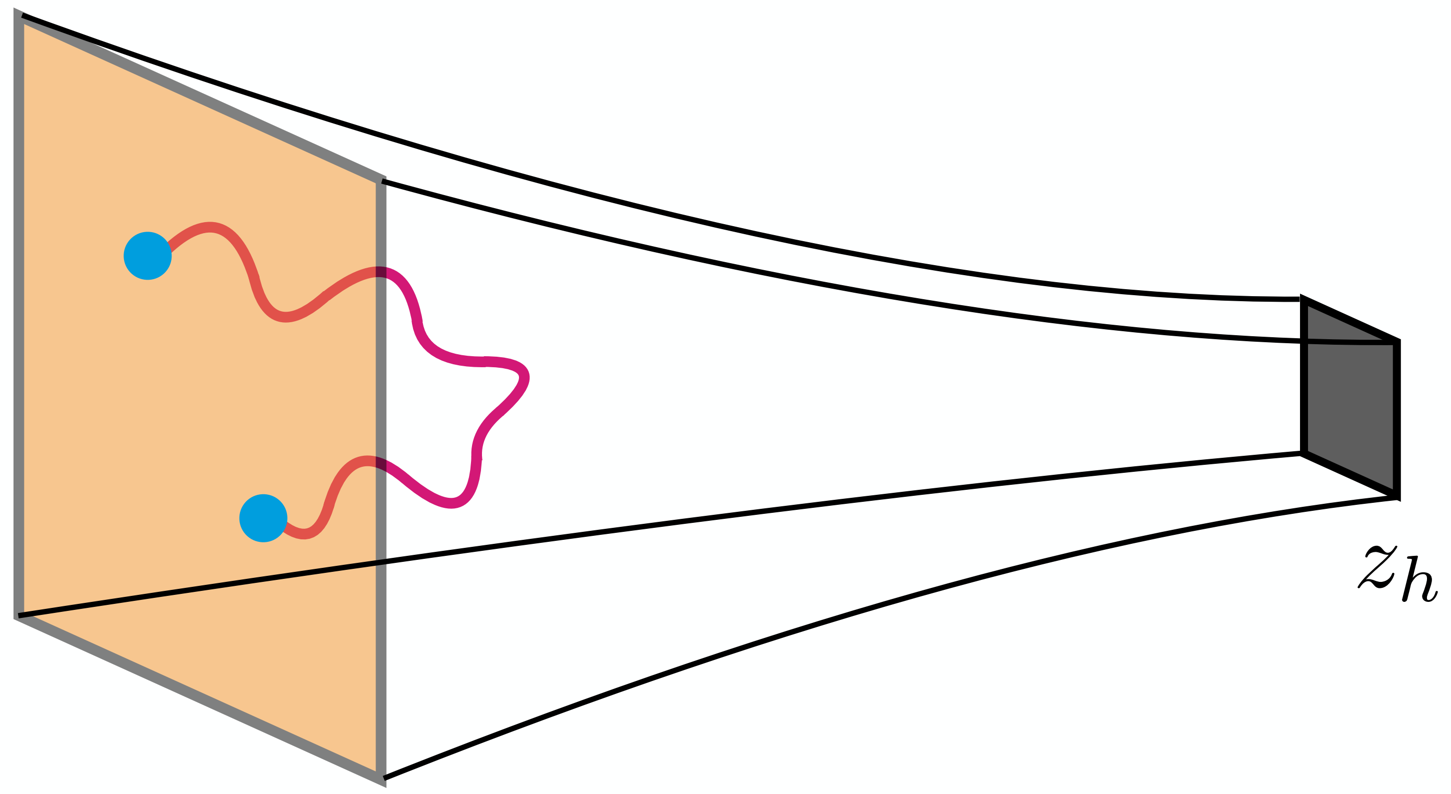

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$[\phi\phi]_{n,J}$ ↓ **double-trace**

$[T^n]$ ↓ **multi-stress-tensor (gravitons)**

3. Applications

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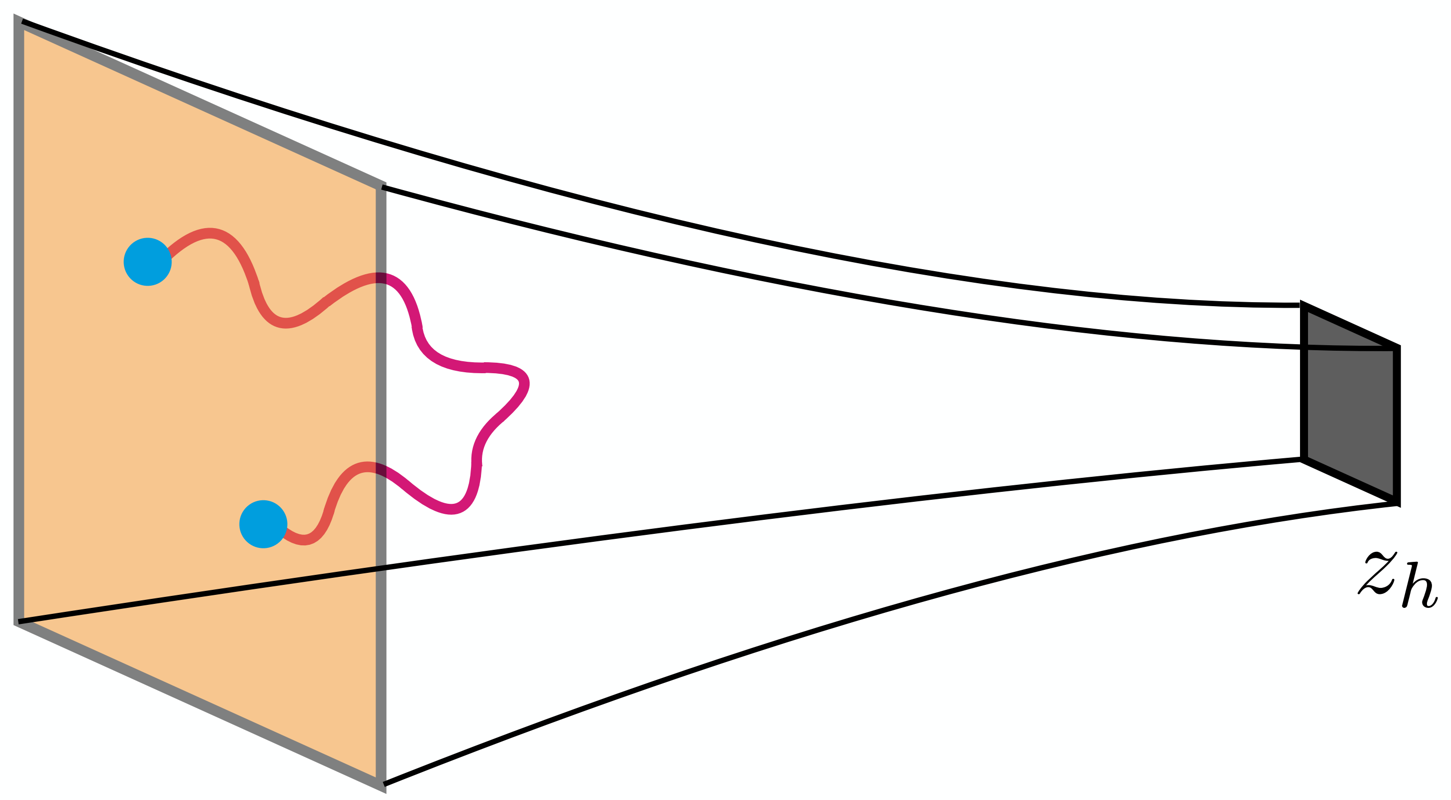

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- Some $[T^n]$ have **poles** for integer Δ_ϕ: $a_{\Delta=8,J=4} = \dfrac{\pi^8 \Delta_\phi(7\Delta_\phi^2 + 6\Delta_\phi + 4)}{201600(\Delta_\phi - 2)}$

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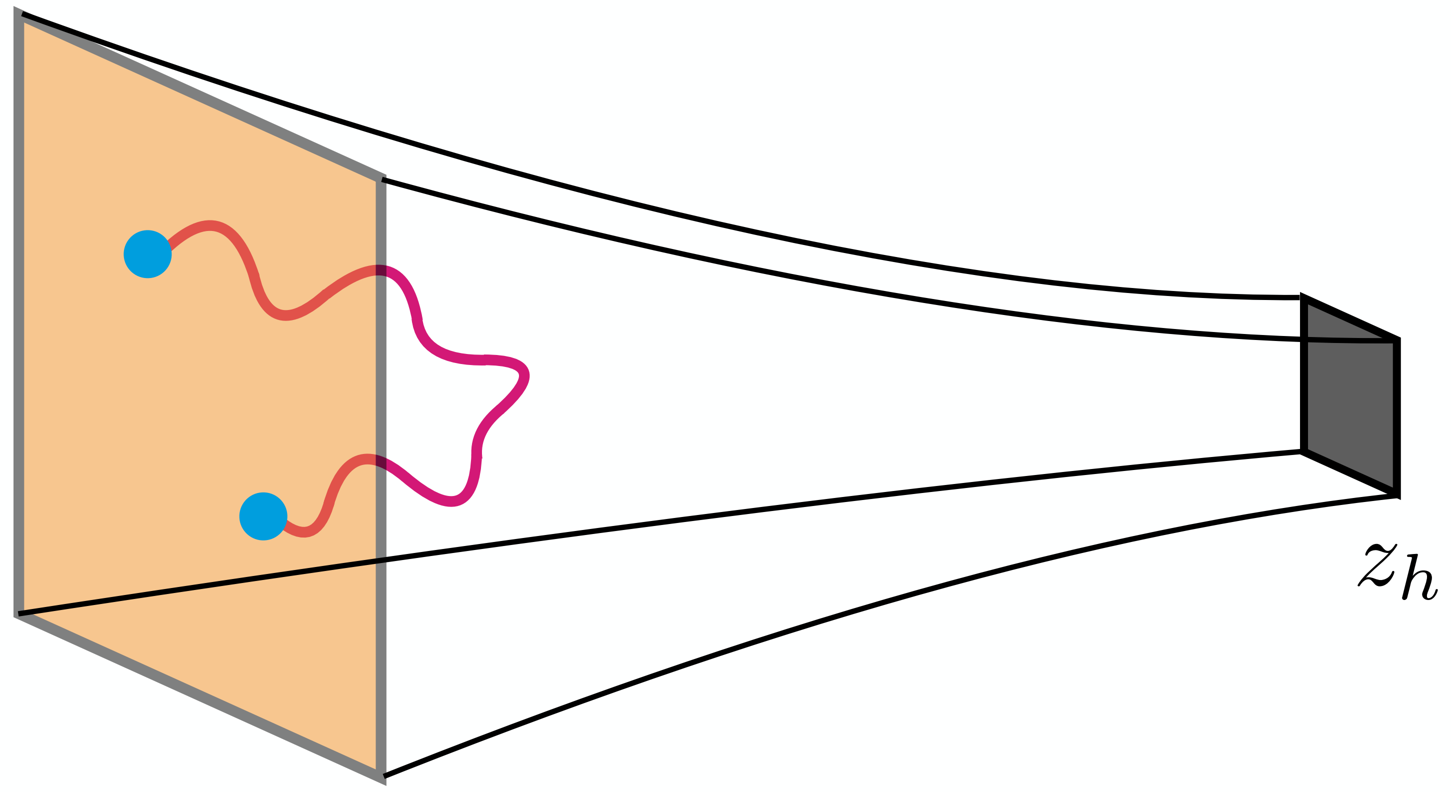

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- Even at large N, **any multi-trace** can in principle contribute

  → Spectrum: $\phi\times\phi \longrightarrow \mathbb{1} + [\phi\phi]_{n,J} + [T^n]$

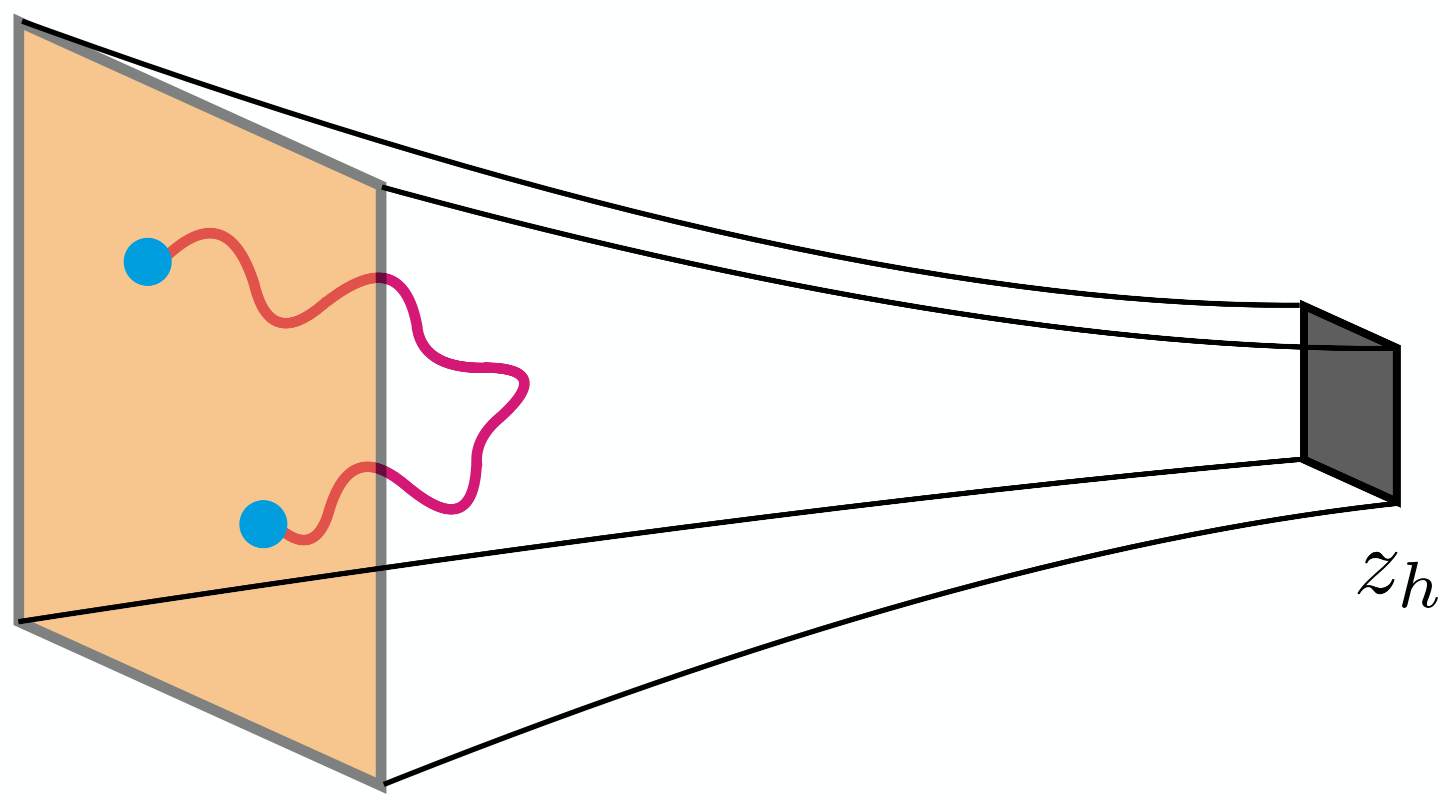
- Some $[T^n]$ have **poles** for integer Δ_ϕ: $\quad a_{\Delta=8,J=4} = \dfrac{\pi^8\Delta_\phi(7\Delta_\phi^2+6\Delta_\phi+4)}{201600\boxed{(\Delta_\phi-2)}}$

3. Applications

4*d* HOLOGRAPHIC THEORIES

[JB, Bozkurt, Marchetto, Miscioscia, Pomoni, WIP]

A long standing problem: $\langle\, \phi(0,0)\phi(\tau,x)\,\rangle_\beta$ at $\lambda = \infty$ for Δ_ϕ integer $(\mathcal{N} = 4$ SYM$)$



[Fitzpatrick, Huang, ’19]
[Dodelson, Grassi, Iossa, Lichtig, Zhiboedov, ’22]
[Parisini, Skenderis, Withers, ’23]
[Burić, Gusev, Parnachev, '25]

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  ⟶ Must be cancelled by poles in (degenerate) $[\phi\phi]_{n,J}$

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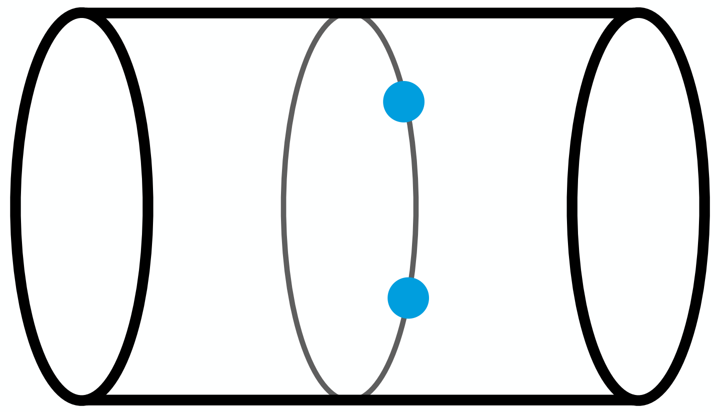


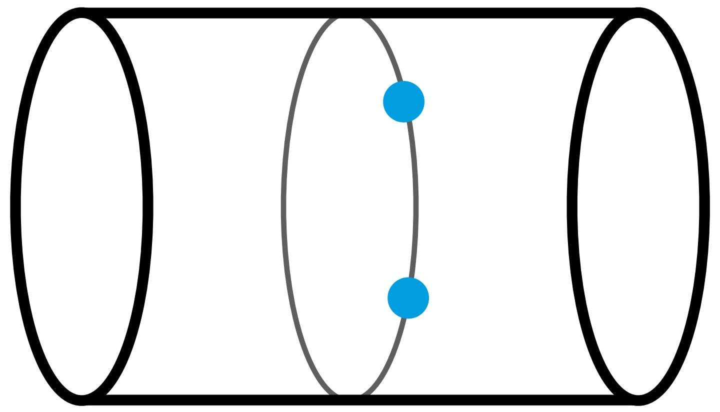
3. Applications

4*d* HOLOGRAPHIC THEORIES

[JB, Bozkurt, Marchetto, Miscioscia, Pomoni, WIP]

Degeneracies: An advantage for analytic bootstrap!

Bootstrap $x = 0$: • $g(\tau) = \sum_{n=0}^{\lfloor(\Delta_\phi - 1)/2\rfloor} a_{T^n}\, g_{\text{GFF}}(2\Delta_\phi - \Delta_{T^n}, \tau) + \kappa$



→ $[\phi\phi]$ **not contributing**, only $[T^n]$ **without poles**!

3. Applications

4d HOLOGRAPHIC THEORIES

[JB, Bozkurt, Marchetto, Miscioscia, Pomoni, WIP]

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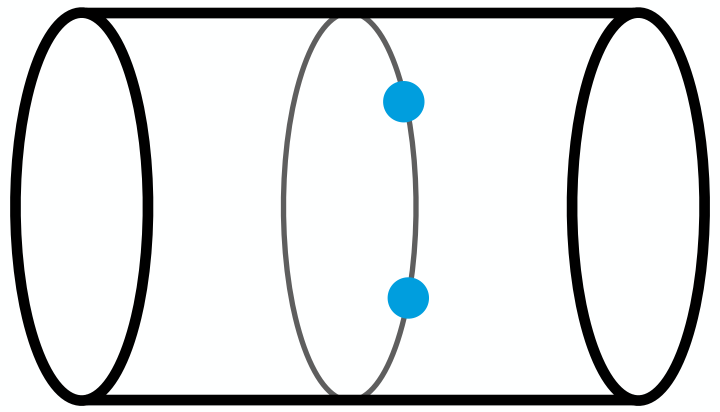

$\longrightarrow$ $[\phi\phi]$ **not contributing**, only $[T^n]$ **without poles**!

- $$g(\tau) = \pi^{2\Delta_\phi} \csc^{2\Delta_\phi}\left(1 - \Theta(\Delta_\phi - 1)\frac{\Delta_\phi}{3}\sin^2(\pi\tau) + \Theta(\Delta_\phi - 2)\frac{\Delta_\phi(\Delta_\phi - 7)}{72}\sin^4(\pi\tau) - \Theta(\Delta_\phi - 3)\frac{\Delta_\phi(28\Delta_\phi^2 - 147\Delta_\phi + 230)}{4536}\sin^6(\pi\tau) + \dots\right) + \kappa_{\Delta_\phi}$$

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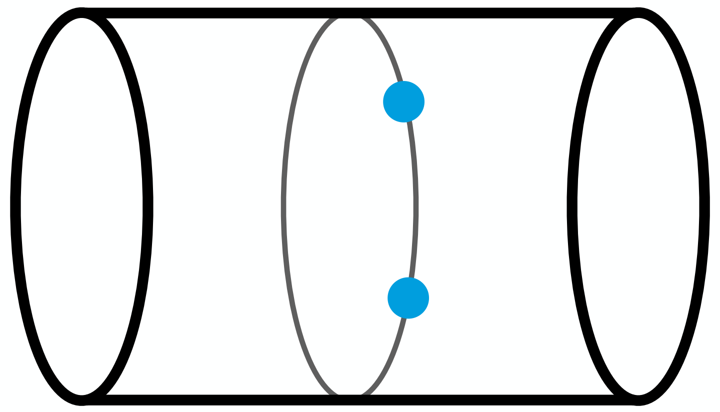

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Bootstrap $x \neq 0$: Work in progress (Arcs are not trivial)

4. Outlook

4. Outlook

INTERESTING DIRECTIONS

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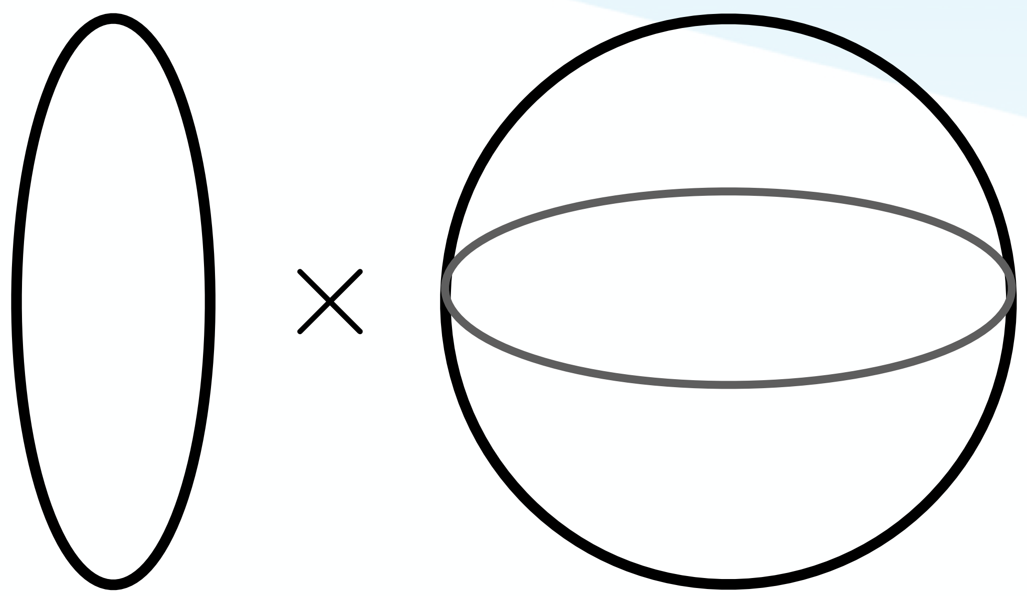
- Phase transitions on $S^1_\beta \times S^{d-1}_R$



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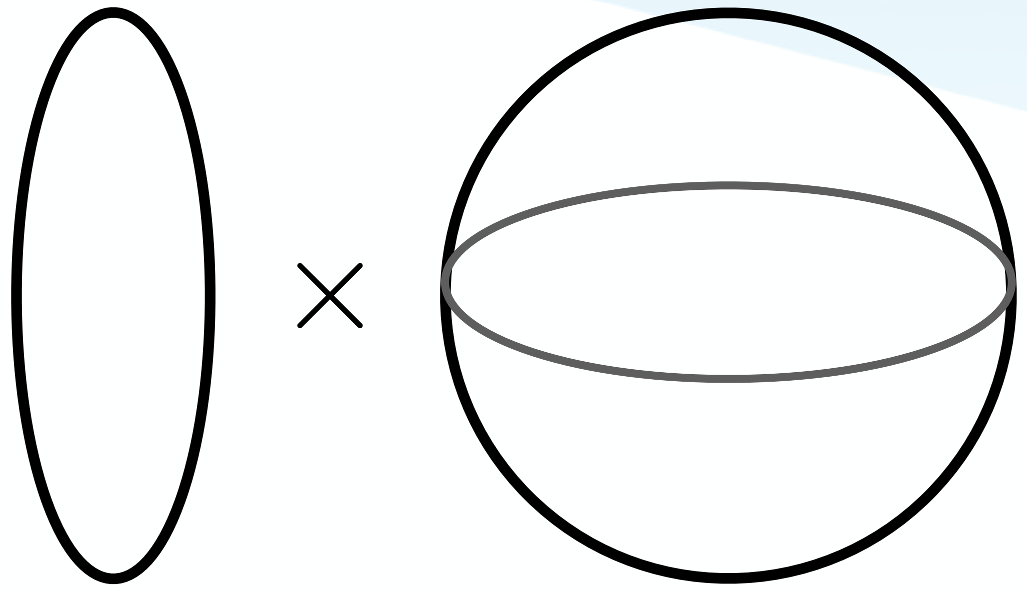

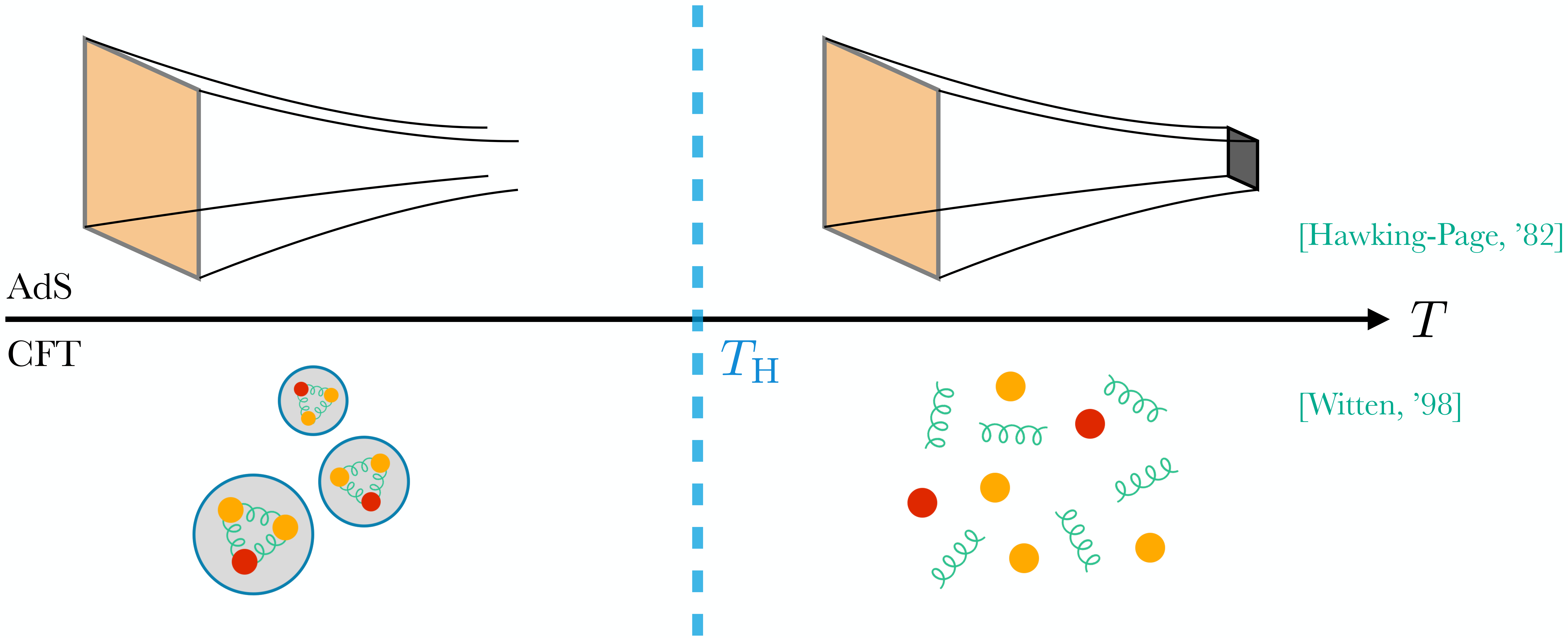

4. Outlook

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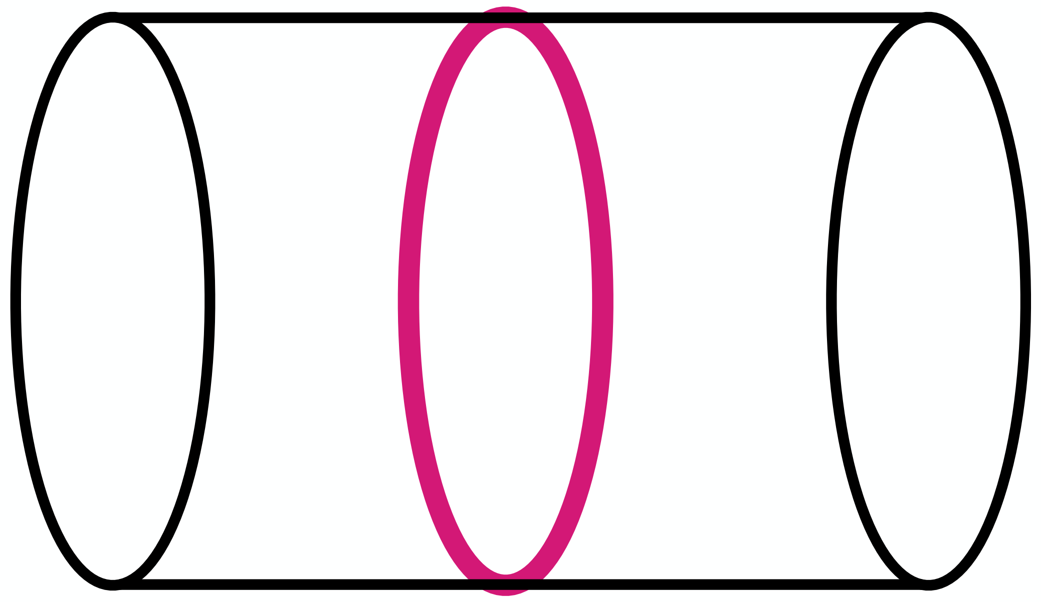

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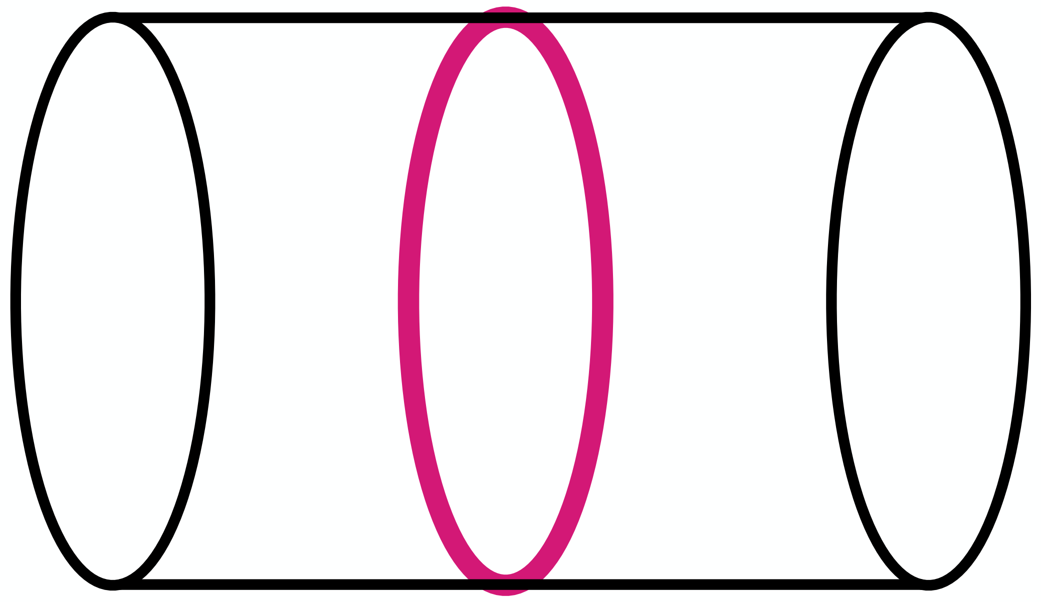

- Polyakov loops (and other defects)
    - Previous work:

	no defect	defect
$T=0$	$\Delta_{\mathcal{O}}$ $f_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3}$	$\hat{\Delta}_{\hat{\mathcal{O}}}$ $\hat{f}_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2\hat{\mathcal{O}}_3}$ $\lambda_{\mathcal{O}\hat{\mathcal{O}}}$
$T\neq 0$	$b_{\mathcal{O}}$	$\boxed{\hat{b}_{\hat{\mathcal{O}}}}$

[JB, Fiol, Marchetto, Miscioscia, Pomoni, '24]

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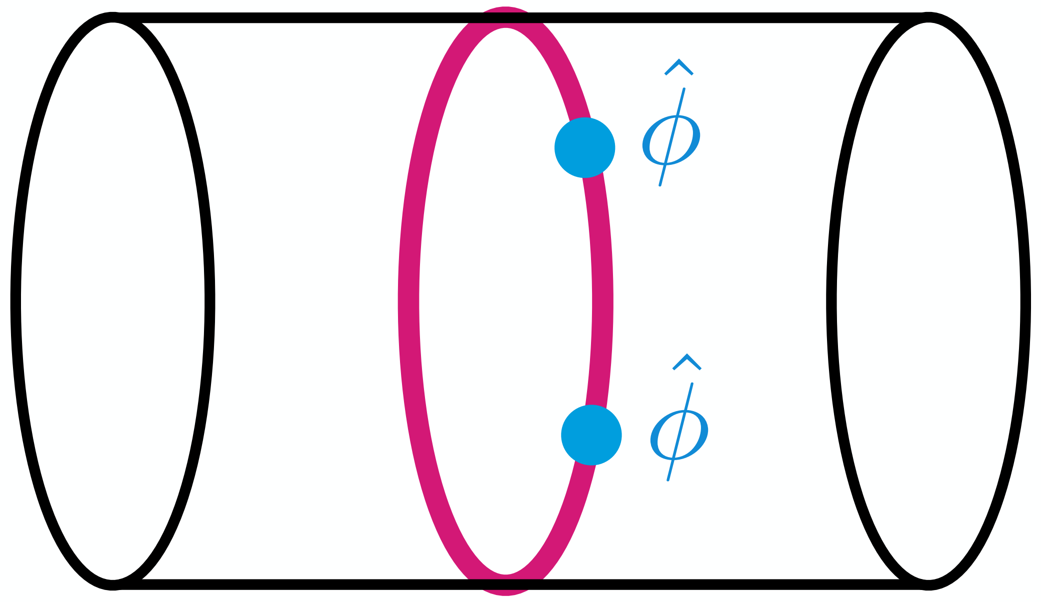

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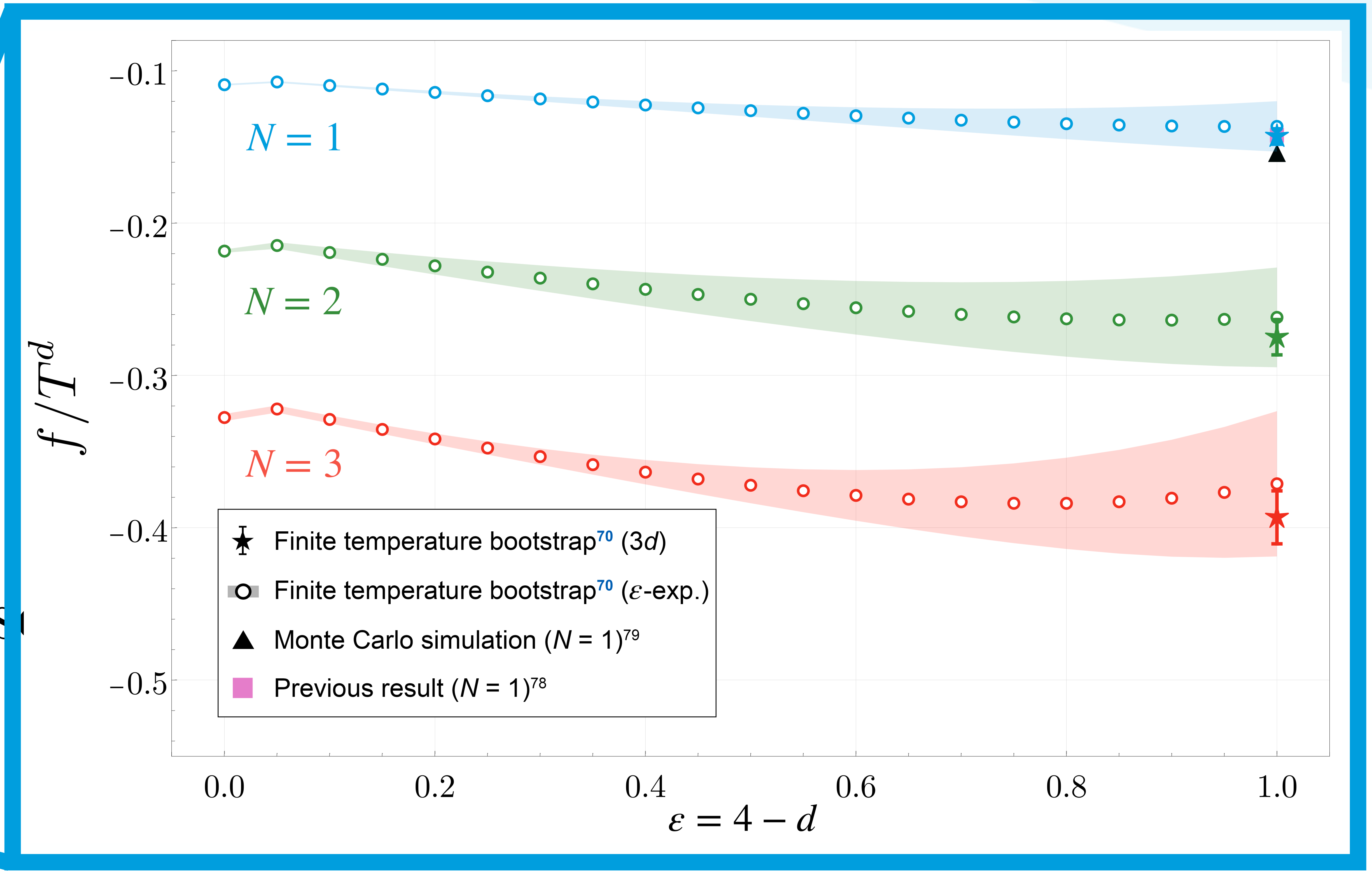

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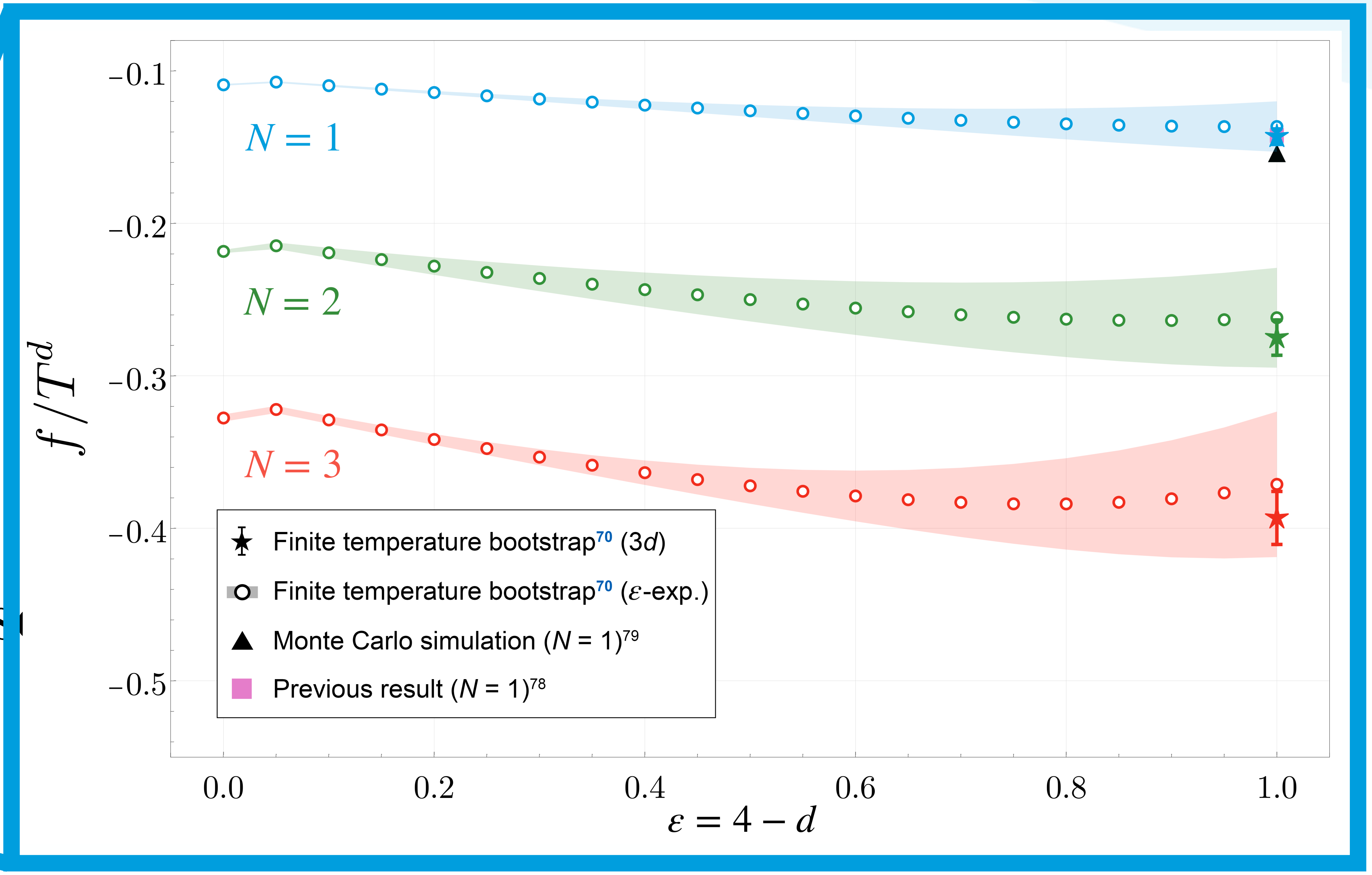

(Sum rules + low spectrum + asymptotics)

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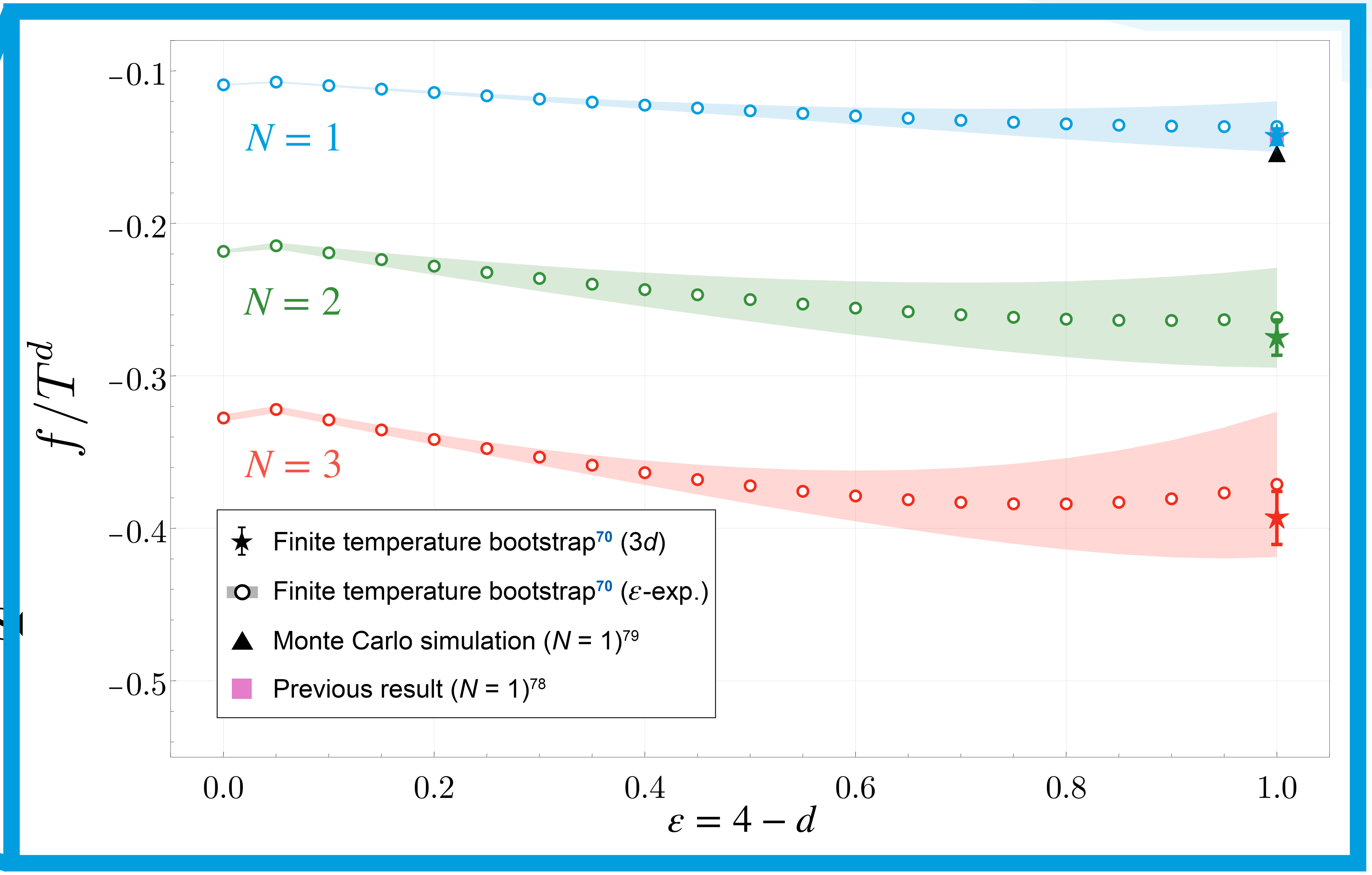

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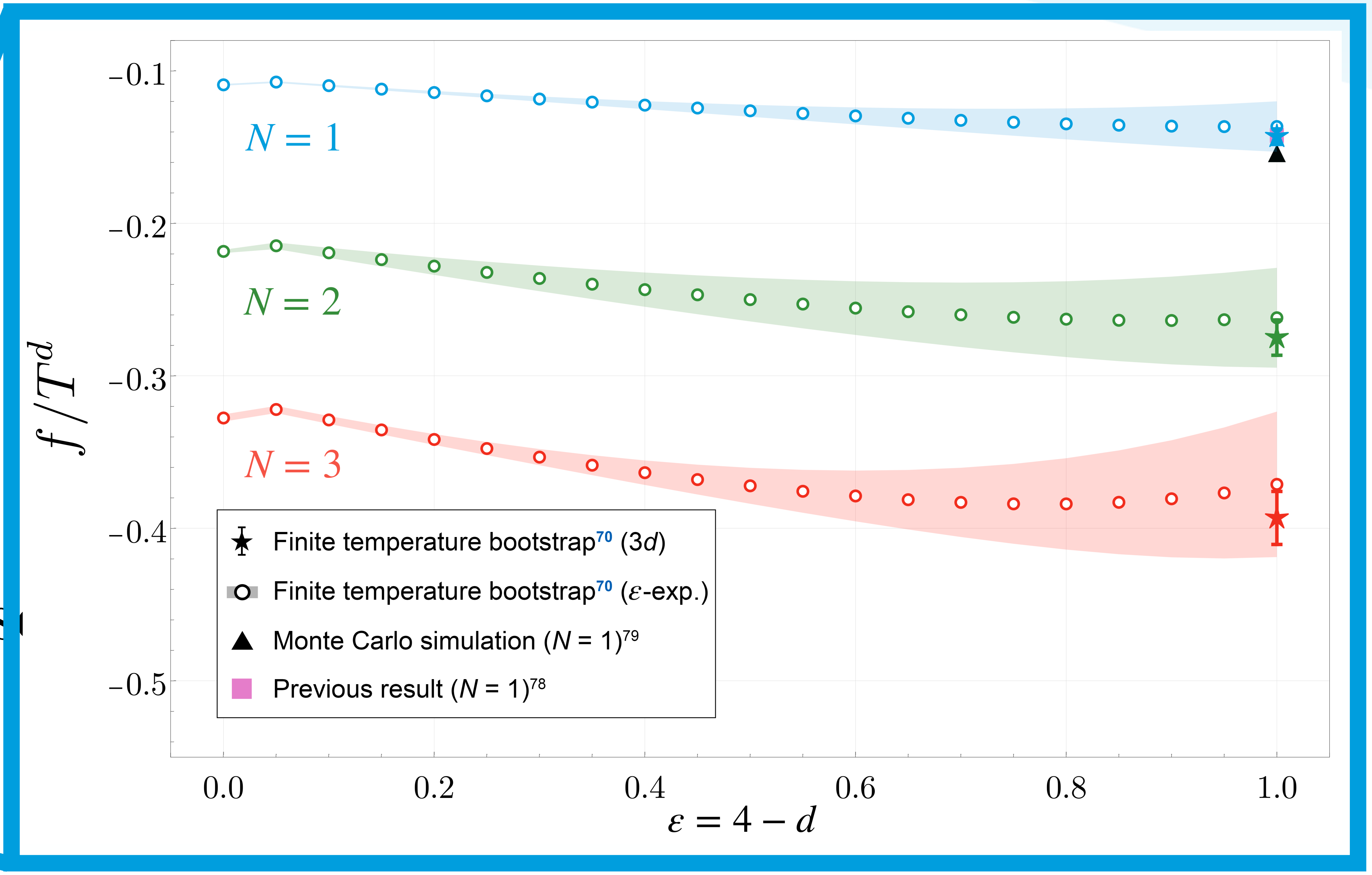

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✓ ✓

[Marchetto, Miscioscia, Pomoni, '23]

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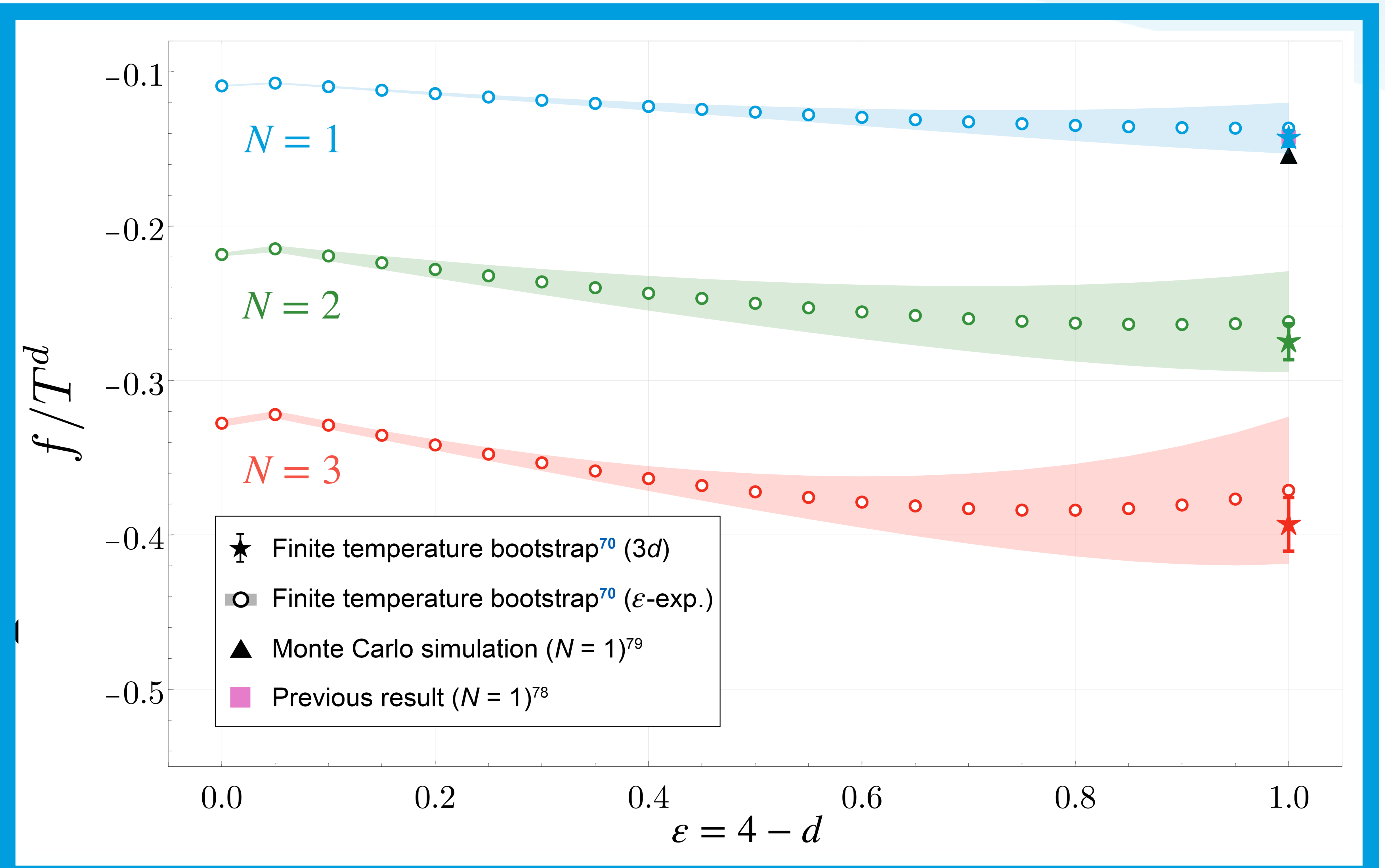

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[Marchetto, Miscioscia, Pomoni, '23] [Gromov, Hegedus, Julius, Sokolova, '23]
[Chester, Dempsey, Pufu, '23]

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THANK YOU!