Virasoro-Shapiro amplitudes in AdS

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Based on: arXiv:2412.06429, with D. Zhong
arXiv:2412.08689 with T. Hansen and D. Zhong
and earlier work

Virasoro-Shapiro amplitude

- The VS amplitude is the most basic quantity in string theory:

$$A^{(0)}(S,T) = \frac{\Gamma(-S)\Gamma(-T)\Gamma(-U)}{\Gamma(S+1)\Gamma(T+1)\Gamma(U+1)}$$

  - $S = -(p_1+p_2)^2\ell_s^2/4\,, \quad T = -(p_1+p_3)^2\ell_s^2/4\,, \quad U = -S-T$ are Mandelstam variables in terms of 10d momenta.

- Describes scattering of closed strings at tree level (leading string coupling g_s), but at finite energy (string length ℓ_s).

- Computed using worldsheet as 2d integral:

$$A^{(0)}(S,T) = -\frac{1}{3U^2}\int d^2z|z|^{-2S-2}|1-z|^{-2T-2} + \text{crossed}\,.$$

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Properties of VS

- VS has poles in S, T, U that correspond to massless graviton multiplet, as well as tower of massive $O(\ell_s^{-2})$ string modes.
- In the high energy limit $S, T \to \infty$ with fixed S/T [Gross, Mende '88] :

$$A^{(0)}(S,T) \sim e^{-2S\log|S|-2T\log|T|-2U\log|U|}$$

  - Exponentially softer than particle scattering.
- Low energy expansion $S, T \to 0$:

$$A^{(0)}(S,T) = -1/(STU) - 2\zeta(3) + (S^2+T^2+U^2)\zeta(5) + \dots$$

  - Leading is graviton multiplet exchange, polynomial corrections are contact terms from higher derivative corrections D^nR^4.

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String theory in AdS

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  - IIB string theory on $AdS_5 \times S^5 \Leftrightarrow$ 4d $\mathcal{N} = 4$ $SU(N)$ SYM.
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- Closed string scattering $\Leftrightarrow$ correlator of stress-tensor multiplet.
  - Tree level in string theory $\Leftrightarrow$ leading $1/N$ in CFT.
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  - AdS backgrounds involve RR flux, worldsheet not understood well in that case (tho hope from pure spinor and string field theory).
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This talk

- Bootstrap AdS VS in expansion around flat space for each AdS/CFT pair using two assumptions:
  1. Amplitude expanded in AdS around flat space given by flat space limit [Penedones '10] (i.e. Borel transform) of CFT correlator.
  2. Amplitude given by worldsheet integral of certain single valued multiple polylogarithms (SVMPLs).
- Originally for AdS_5/CFT_4 [Alday, Hansen, Silva '22], and now AdS_4/CFT_3 [SMC, Hansen, Zhong '24] and AdS_3/CFT_2 [SMC, Zhong '24].
- Extract data of single-trace operators, compare to integrability!

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Holographic correlator

- Consider CFT operator $\mathcal{O}$ dual to graviton mode in AdS.
- Expand correlator of $\mathcal{O}$'s in superblocks $G_{\Delta,\ell}$:

$$\langle\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{1}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}} \sum_{\Delta,\ell} G_{\Delta,\ell}(U,V) + \text{prot}$$

  - $U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$ and $V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$ are conformal cross ratios.
  - $G_{\Delta,\ell}$ for each unprotected supermultiplet, are linear combos of conformal blocks as fixed by superconformal symmetry.
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Large N

- Write correlator $H(U, V)$ in Mellin space ($u + s + t = 4\Delta_{\mathcal{O}}$):

$$H(U,V) = \int \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}-\Delta_{\mathcal{O}}} \Gamma\left[\Delta_{\mathcal{O}} - \frac{s}{2}\right]^2 \Gamma\left[\Delta_{\mathcal{O}} - \frac{t}{2}\right]^2 \Gamma\left[\Delta_{\mathcal{O}} - \frac{u}{2}\right]^2 \mathcal{M}(s,t)$$

  - Poles in s, t, u correspond to Δ of exchanged operators.
  - Γ's take into account all double trace poles.

- Planar $\mathcal{M}(s,t) \equiv M(s,t)/N^{\#} + \dots$ (what we consider in this talk):

$$M(s,t) = \text{sugra}(s,t) + \sum_i^{\infty} b_i(\lambda) P_i(s,t)$$

  - sugra is exchange of graviton multiplet, is meromorphic in s, t.
  - P_i are degree i polynomials in s, t, are contact terms from higher derivative terms, $b_i(\lambda)$ depend on AdS radius $(R/\ell_s)^4 \sim \lambda$.

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$$H(U,V)=\int\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}}V^{\frac{t}{2}-\Delta_{\mathcal{O}}}\Gamma\big[\Delta_{\mathcal{O}}-\frac{s}{2}\big]^2\Gamma\big[\Delta_{\mathcal{O}}-\frac{t}{2}\big]^2\Gamma\big[\Delta_{\mathcal{O}}-\frac{u}{2}\big]^2\mathcal{M}(s,t)$$

  - Poles in s,t,u correspond to Δ of exchanged operators.
  - Γ's take into account all double trace poles.

- Planar $\mathcal{M}(s,t)\equiv M(s,t)/N^{\#}+\dots$ (what we consider in this talk):

$$M(s,t)=\text{sugra}(s,t)+\sum_i^{\infty}b_i(\lambda)P_i(s,t)$$

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- Get flat space amplitude $A^{(0)}(S,T)$ from $M(s,t)$ [Penedones '10] :

$$A^{(0)}(S,T) = \lim_{\lambda\to\infty} \int \frac{d\alpha}{2\pi i} e^{\alpha} \alpha^{d/2-2\Delta_{\mathcal{O}}} M\Big(\frac{2\sqrt{\lambda}S}{\alpha}, \frac{2\sqrt{\lambda}T}{\alpha}\Big),$$

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- Also expand tree CFT data (single trace operators, $\delta = 1, 2, \dots$):

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Worldsheet ansatz for AdS correction

- **Assume** that $A^{(1)}(S,T)$ given by worldsheet integral:

$$A^{(1)}(S,T) = \int d^2z |z|^{-2S-2}|1-z|^{-2T-2}\sum_i \frac{p_i(S,T)}{U}\mathcal{L}_i(z) + \text{crossed}\,.$$

- $\mathcal{L}_i(z)$ are weight 3 single-valued multiple polylogs (SVMPLs), bc:
  - Low energy expansion gives single valued zeta functions, e.g. $\zeta(3), \zeta(5)$ (but not $\zeta(2)$), as expected for tree level closed strings.
  - Weight 3 SVMPLs lead to degree 4 poles in S in $A^{(1)}(S,T)$, matches poles in block expansion.
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$\mathcal{N} = 4$ SYM correlator

- Consider correlator of stress tensor multiplet.
- Superblock expansion given by [Dolan, Osborn '02] .
  - Only long multiplets in singlet of $SU(4)_R$ with even spin ℓ.
  - Lowest dimension $\ell = 0$ single trace multiplet called the Konishi.
- Scaling dimensions of spin ℓ single trace operators computed from integrability for first three AdS corrections, i.e. to $1/\lambda^{5/4}$, using quantum spectral curve [Gromov, Levkovich-Maslyuk, Sizov, Valatka '14] .
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- From $A^{(1)}$, match τ of lowest regge trajectory to integrability:

$$\tau(\ell/2+1,\ell) = \lambda^{1/4}\sqrt{2\ell+4} - \ell - 2 + \frac{3\ell^2+10\ell+16}{4\lambda^{1/4}\sqrt{2\ell+4}} + O(\lambda^{-3/4})\,.$$

- Can also get OPE coefficient to same order, matches result from numerical bootstrap+integrability+localization in strong coupling regime [Caron-Huot,Coronado, Zahraee '24] .
- Get low energy expansion of correlator to D^4R^4 order, matches bootstrap+localization. Combine all methods to get unprotected tree $D^8R^4 \sim \zeta(7)$ for first time!

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- Low energy expansion matches protected terms from localization for AdS_4 and AdS_5, entirely new for AdS_3.
  - New prediction of unprotected terms for AdS_4 and AdS_5!

Conclusion

- Fix first correction $A^{(1)}$ to flat space for AdS_3, AdS_4, and AdS_5.
- Prediction of Δ and λ^2 of lowest regge trajectory.
    - For AdS_5 matches results from integrability and bootstrap.
    - For AdS_4 compare $\mathcal{Z}$ even Δ to integrability, but $\mathcal{Z}$ odd Δ and all λ^2 are new predictions.
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Future directions

- Higher order AdS corrections, already have $A^{(2)}$ for AdS_4 and AdS_5 by inputting integrability and localization results.
- Compare predictions to integrability (e.g. $\mathcal{Z}$ odd for AdS_4).
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