News from

4D $\mathcal{N}=2$ SCFT spin chains

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IGST 2025

Funded by the European Union

European Research Council
Established by the European Commission

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[2507.08934 Bozkurt, Nieto García, Kong, EP]

[2408.03365 Bozkurt, Nieto García, EP]

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[2411.11612 Bertle, EP, Zhang, Zoubos]

Motivation

* Is $\mathcal{N}=4$ SYM the only* integrable theory in 4D?

* What happens in 4D when reduce supersymmetry?

Motivation

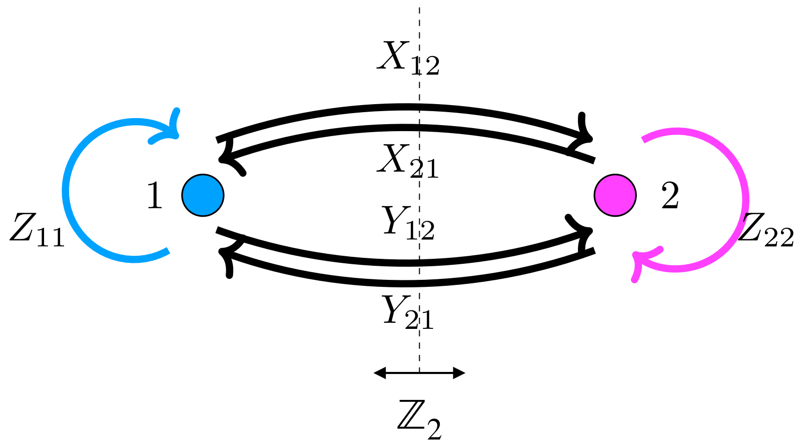
* $\mathcal{N}=2$ SCFTs: the simplest next step.

* Orbifolds of $\mathcal{N}=4$ SYM + marginal deformation: span a big subset of the landscape of Lagrangian $\mathcal{N}=2$ SCFTs.

* Also generalisable to a large class of $\mathcal{N}=1$ SCFTs.

The simplest example

Z2 quiver theory with SU(N)xSU(N) color group



Z2 orbifold $\mathcal{N}=4$ SYM **marginally deformed** from the **orbifold point** ($g_1 = g_2$)

The orbifold projection:

$$X = \begin{pmatrix} 0 & X_{12} \\ X_{21} & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & Y_{12} \\ Y_{21} & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{pmatrix}$$

(bif: X, Y; Adj: Z_{11}, Z_{22})

✱ When $g_2 \to 0$ gives $\mathcal{N}=2$ SCQCD in the Veneziano limit ($N_f = 2N_c$).

In this talk

* Bottom up: Long-range coordinate Bethe ansatz for three- and four-magnon eigenvectors

* Top down: Novel Groupoid symmetry structures

Long-range Coordinate Bethe Ansatz

[2408.03365 Bozkurt, Nieto García, EP]

[2507.08934 Bozkurt, Nieto García, Kong, EP]

The SU(2)$_{XZ}$ sector

- ✱ The SU(2) rotation on XZ is naively broken (by the orbifold)
- ✱ Adjoint vacua: ZZZ: $\cdots Z_{11}Z_{11}Z_{11}\cdots$ and $\cdots Z_{22}Z_{22}Z_{22}\cdots$
- ✱ Bifundamental vacua: XXX: $\cdots X_{12}X_{21}X_{12}\cdots$
- ✱ In this talk: only the adjoint ZZZ vacua
- ✱ Excitations around XXX have very different behaviour

$$E(p) = \kappa + \frac{1}{\kappa} - \sqrt{\left(\kappa + \frac{1}{\kappa}\right)^2 - 2sin^2p}$$

$\kappa = \frac{g_2}{g_1}$

[2106.08449 EP, Rabe, Zoubos]

One magnon

One bifundamental X excitation interpolates between the two adjoint vacua:

$$|\Psi(p)\rangle_{21} = \sum_{x=-\infty}^{\infty} e^{ipx} | \cdots Z_{22} Z_{22} X_{21}(x) Z_{11} Z_{11} \cdots \rangle$$

$$|\Psi(p)\rangle_{12} = \sum_{x=-\infty}^{\infty} e^{ipx} | \cdots Z_{11} Z_{11} X_{12}(x) Z_{22} Z_{22} \cdots \rangle$$

$$\mathbb{Z}_2 |\Psi(p)\rangle_{ij} = |\Psi(p)\rangle_{ji} \,, \quad i,j = 1,2$$

Energy eigenvalue

$$E(p) = \left(\sqrt{\kappa} - \frac{1}{\sqrt{\kappa}} \right)^2 + 4 \sin^2 \left(\frac{p}{2} \right)$$

$$\kappa = \frac{g_2}{g_1}$$

[1006.0015 Gadde, EP, Rastelli]

Together parity and $\mathbb{Z}_2$: reading the chain backwards $\mathscr{P}\mathbb{Z}_2 |\Psi(p)\rangle_{ij} = |\Psi(-p)\rangle_{ji}$

Two magnons

Two bifundamental excitations

$$|\Psi(p_1,p_2)\rangle_{11} = \sum_{x_1<x_2} \psi_{11}(p_1,p_2;x_1,x_2)\,|\cdots Z_{11}X_{12}(x_1)Z_{22}\cdots Z_{22}X_{21}(x_2)Z_{11}\cdots\rangle$$

$$|\Psi(p_1,p_2)\rangle_{22} = \sum_{x_1<x_2} \psi_{22}(p_1,p_2;x_1,x_2)\,|\cdots Z_{22}X_{21}(x_1)Z_{11}\cdots Z_{11}X_{12}(x_2)Z_{22}\cdots\rangle$$

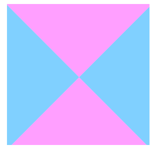
$\mathbb{Z}_2$ conjugate

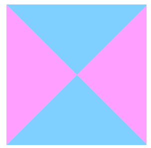
With $\psi_{ii}(p_1,p_2;x_1,x_2) = (e^{ix_1p_1+ix_2p_2} + S_{ii}(p_1,p_2)e^{ix_1p_2+ix_2p_1})\,, \quad i = 1{,}2$

Energy eigenvalue $E_2(p_1,p_2) = E_1(p_1) + E_1(p_2)$

Scattering coefficients

$$S_\kappa(p_1,p_2) = -\frac{1+e^{ip_1+ip_2}-2\kappa e^{ip_2}}{1+e^{ip_1+ip_2}-2\kappa e^{ip_1}}$$

$S_{11} = S_\kappa(p_1,p_2) =$ 

$S_{22} = S_{1/\kappa}(p_1,p_2) =$ 

[1006.0015 Gadde, EP, Rastelli]

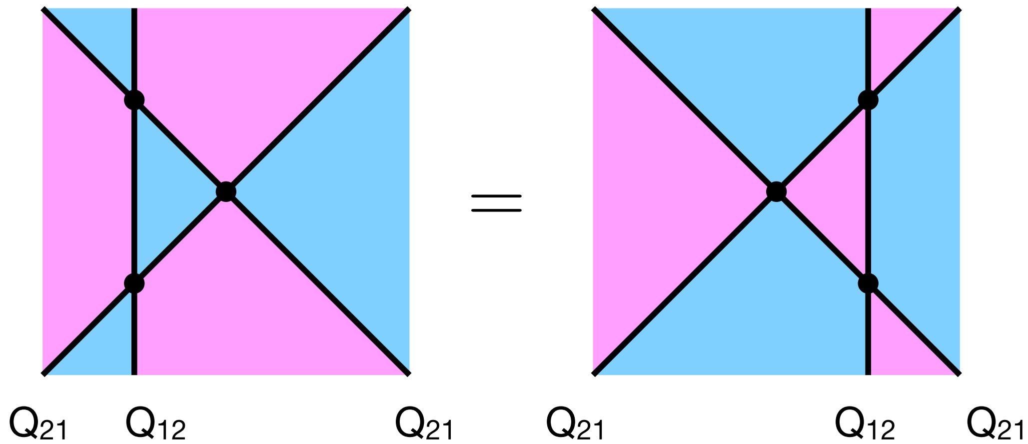
Three magnons

- ✱ No solution with standard coordinate Bethe ansatz and

$$E_3(p_1, p_2, p_3) = \sum_{n=1,2,3} E_1(p_n)$$

- ✱ Even when we add (a finite number of) contact terms to the CBA.

- ✱ Precisely because the naive YBE is not satisfied.



$$S_{1/\kappa}(p_2, p_3)\, S_\kappa(p_1, p_3)\, S_{1/\kappa}(p_1, p_2) \;\neq\; S_\kappa(p_1, p_2)\, S_{1/\kappa}(p_1, p_3)\, S_\kappa(p_2, p_3)$$

Three & Four magnons

The only way to get an eigenvector with 3 or more excitations:

Is to allow for **infinite** position dependent corrections to the CBA:

$$\Psi_3(\overrightarrow{p}; \overrightarrow{x}) = \left(\sum_{\sigma \in S_3} \left(A_\sigma + D_\sigma^{n,m}\right) e^{i \overrightarrow{p}_\sigma \cdot \overrightarrow{x}} \right)$$

$$\Psi_4(\overrightarrow{p}; \overrightarrow{x}) = \left(\sum_{\sigma \in S_3} \left(A_\sigma + D_\sigma^{n,m,r}\right) e^{i \overrightarrow{p}_\sigma \cdot \overrightarrow{x}} \right)$$

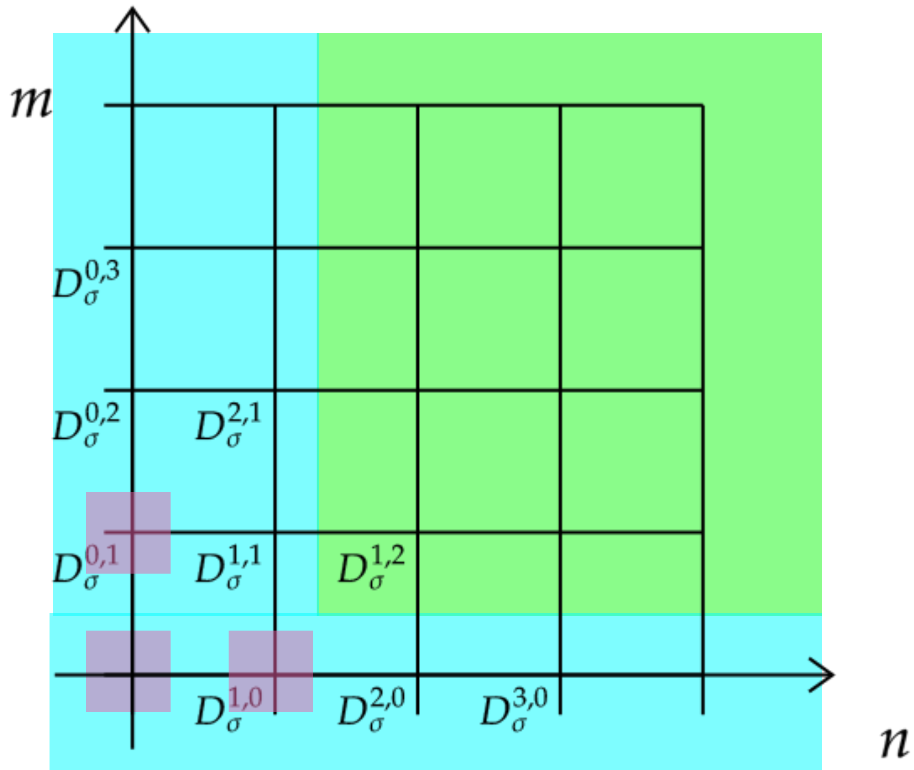
With the integers: $n = x_2 - x_1 - 1$, $m = x_3 - x_2 - 1$ and $r = x_4 - x_3 - 1$ labelling the distances between the 3 or 4 magnons respectively.

A lattice of corrections

For three magnons the $D_\sigma^{n,m}$ corrections to the CBA arrange on a lattice with nearest neighbor recursion relations:

Eigenvalue equations:

* non-interacting
* two-magnon interacting
* three-magnon interacting



For four magnons the $D_\sigma^{n,m,r}$ arrange on a cube.

Special solution

- Only imposing the eigenvalue equations, does not completely fix all coefficients.

- Makes sense: as we have not yet imposted any BC.

- To further fix unknown coefficients: make sure that the $\mathbb{Z}_2$ **symmetry** of the orbifold is **manifest** (at the level of eigenvectors) as well as, that

- For four magnons and short periodic spin chains (with untwisted or twisted BC) we get the same answer as direct diagonalisation (we have checked up to L=6).

Impose:

$$A_\sigma = \prod_{i<\sigma(i)} S_\kappa(p_i, p_{\sigma(i)})\,, \quad D^{n,m}_\sigma = S_\kappa(p_{\sigma(1)}, p_{\sigma(2)})\, D^{n,m}_{(\sigma(1)\sigma(2))\,\sigma}$$

Special solution

↓ Three magnon interacting eqn.

$$D_\sigma^{0,1} = D_\sigma^{1,0} = \left(\kappa - \frac{1}{\kappa}\right) \prod_{i<\sigma(i)} S_\kappa(p_i, p_{\sigma(i)})$$

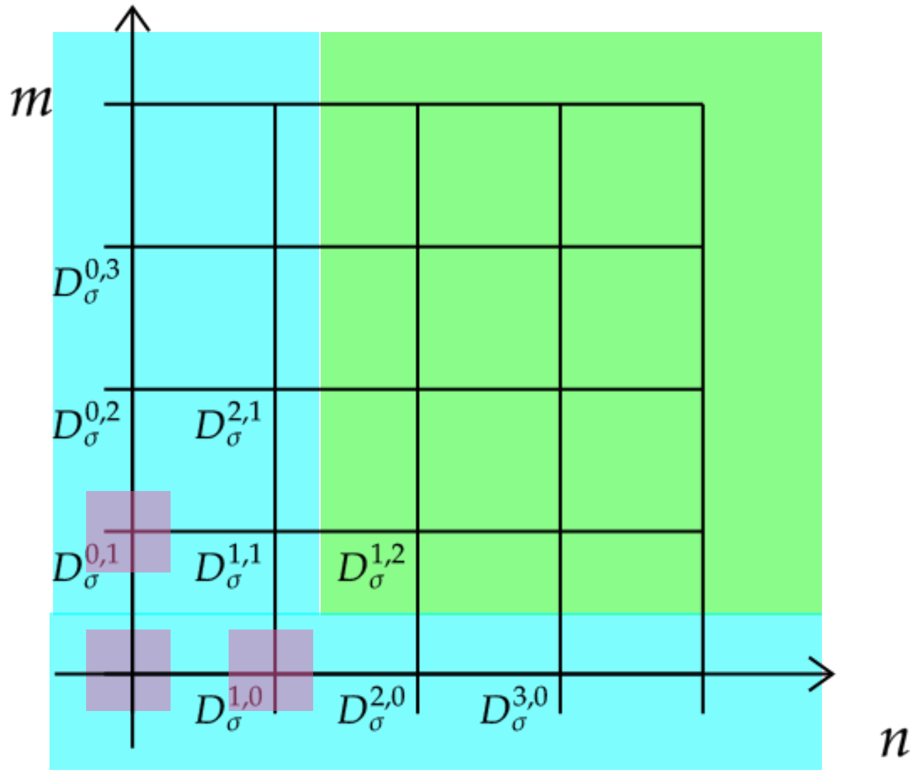
↓ Two magnon interacting eqn.

$$D_\sigma^{1,1} = 0$$

$$D_\sigma^{0,2} = \left(\varepsilon_3 - e^{i\mathscr{P}} - 2\kappa\right)\left(\kappa - \frac{1}{\kappa}\right) \prod_{i<\sigma(i)} S_\kappa(p_i, p_{\sigma(i)})$$

↓ Non-interacting eqn.

$$D_\sigma^{n\geq 1, m\geq 2} = (\cdots)\left(\kappa - \frac{1}{\kappa}\right) \prod_{i<\sigma(i)} S_\kappa(p_i, p_{\sigma(i)})$$



All the corrections are **completely fixed** and they take the form:

$$D_\sigma^{n,m} = \left(\kappa - \frac{1}{\kappa}\right) \prod_{i<\sigma(i)} S_\kappa(p_i, p_{\sigma(i)})\, \hat{f}(p_1, p_2, p_3; n, m) = -\oint \frac{dxdy}{4\pi^2} \frac{G_\sigma(x,y)}{x^{n+1} y^{m+1}}$$

Infinite tower of Yang-Baxter

* The scattering coefficients of the special solution $A_\sigma = \prod S_\kappa(p_i, p_{\sigma(i)})$ factorise and obey the Yang-Baxter equation.

* Remarkably, also the corrections $D_\sigma^{n,m}$ obey **infinite tower of Yang-Baxter like equations** $\forall n, m$, for three-magnons (and also $\forall r$ four-magnons).

$$Y_j^{n,m}(p_2,p_3,p_1)Y_{j+1}^{n,m}(p_1,p_3,p_2)Y_j^{n,m}(p_1,p_2,p_3) = Y_{j+1}^{n,m}(p_1,p_2,p_3)Y_j^{n,m}(p_1,p_3,p_2)Y_{j+1}^{n,m}(p_2,p_3,p_1)$$

The Yang operator captures permutations between different coefficients:

$$Y_j^{n,m}(p_1,p_2,p_3) = \begin{pmatrix} S_\kappa(p_1,p_2)\frac{1+f(p_2,p_1,p_3;n,m)}{1+f(p_1,p_2,p_3;n,m)} & 0 \\ 0 & S_\kappa(p_2,p_1)\frac{1+f(p_3,p_1,p_2;m,n)}{1+f(p_3,p_2,p_1;m,n)} \end{pmatrix} \qquad Y_j^{n,m}(p_1,p_2,p_3)Y_j^{n,m}(p_2,p_1,p_3) = \mathbf{1}$$

Smearing a magnon

Relate an $(M+1)$-magnon eigenvector to an M-magnon eigenvector:

$$\lim_{\bar{p}_4 \to 0} \lim_{L \to \infty} \frac{1}{L} \sum_{x_4 = x_3+1}^{L} \Psi^{(4)}_{11}(p_1, p_2, p_3, p_4, x_1, x_2, x_3, x_4) = \Psi^{(3)}_{12}(p_1, p_2, p_3, x_1, x_2, x_3)$$

At the level of the generating function:

$$G^{(4)}_{\sigma}(x, y, z) \sim \frac{1}{1 - e^{ip_{\sigma(4)}} z} G^{(3)}_{\sigma}(x, y) \,, \quad z \to 1$$

$$G^{(4)}_{\sigma}(x, y, z) \sim \mathit{finite} \,, \quad y \to 1$$

Putting together the **pole structure**: can write the four-magnon generating function as a function of the three-magnon one:

$$\hat{G}^{(4)}_{ijkl}(x, y, z) = \frac{2xyz\left(\kappa - \kappa^{-1}\right)\left(1 - e^{ip_l} z - e^{-ip_i} x\right)}{(1 - e^{ip_l} z)(1 - e^{-ip_i} x)} + \frac{r_L\, \hat{G}^{(3)}_{ijk}(x,0) + \cdots}{(1 - e^{ip_l} z)} + \frac{r_R\, \hat{H}^{(3)}_{jkl}(y,0) + \cdots}{(1 - e^{-ip_i} x)} + \hat{C}_{ijkl}(x, y, z)$$

Why is this happening?

Novel

Groupoid Symmetries

[2411.11612 Bertle, EP, Zhang, Zoubos]

The Hilbert space

$\mathcal{N}=4$ SYM spin chain states: on the lattice sites a “single letter”:

unique ultrashort **singleton representation** of $PSU(2,2|4)$

$$\mathcal{V} = (X, Y, Z, \bar{X}, \bar{Y}, \bar{Z}, \ldots) : \mathbf{adj}_G$$

All single letters are in the **adjoint representation** of the color group G.

The total space is $\otimes_\ell^L \mathcal{V}_\ell$.

$\mathcal{N}=2$ SCFTs spin chain states: *two distinct* ultrashort reps of $SU(2,2|2)$:

$$\mathcal{V} = (Z, \bar{Z}, \ldots) : \mathbf{adj}_G, \quad \mathcal{H} = (X, Y, \bar{X}, \bar{Y}, \ldots) : \mathbf{bif}_{G_1 \times G_2}$$

In the **adjoint** and **bifundamental reps** of the color group $G_1 \times G_2 \times \ldots$

The total space is not $\otimes_\ell^L \mathcal{V}_\ell$. (crossed out)

The Hilbert space

The **color index structure** imposes **restrictions on the total space**!

$X_{12}X_{21}$ allowed, $X_{12}X_{12}$ not allowed, $Z_{11}X_{12}$ allowed, $Z_{22}X_{12}$ not allowed!

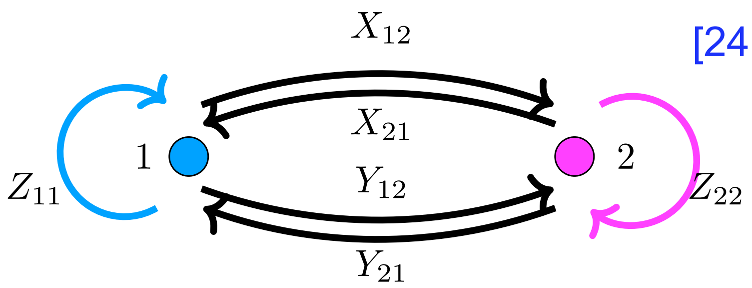
In [2106.08449 Rabe,EP,Zoubos] we identified this structure with a **dynamical spin chain:**

start with $\mathcal{N}$=4 SYM states and specify the first color index:

$$XXZXZZ \to X_{12}X_{21}Z_{11}X_{12}Z_{22}Z_{22} \text{ and } X_{21}X_{12}Z_{22}X_{21}Z_{11}Z_{11}$$

Most elegantly described as a **path groupoid** (follow the arrows).

[2411.11612 Bertle,EP,Zhang,Zoubos]



Matches the 1-category structure in [2010.01060 Felder,Ren Quantum Groups for RSOS]

R-symmetry algebroid

The SU(2)$_{XZ}$ sector

✱ $\mathcal{N}$=4 SYM the SU(2) is **unbroken**: $\begin{pmatrix} Z \\ X \end{pmatrix} = \mathbf{2}$ its doublet rep.

✱ $\mathcal{N}$=2 orbifold the SU(2) is **broken**: $\begin{pmatrix} Z_{11} \\ X_{12} \end{pmatrix}$ or $\begin{pmatrix} Z_{22} \\ X_{21} \end{pmatrix}$ cannot be doublets

as they are in different color reps.

✱ The broken generators can be recovered by **moving beyond the Lie algebraic** setting to that of a **Lie algebroid**.

The SU(2)$_{XZ}$ sector

First for ***single letter*** words.

* $\mathcal{N}=4$ SYM unbroken SU(2): we have raising/lowering generators:

$$\sigma_+ Z = X \text{ and } \sigma\_X = Z$$

* $\mathcal{N}=2$ SCFT broken SU(2): define **two copies** of raising/lowering:

$$\sigma_+^{(1)} Z_{11} = X_{12} \text{ and } \sigma_-^{(1)} X_{12} = Z_{11}$$

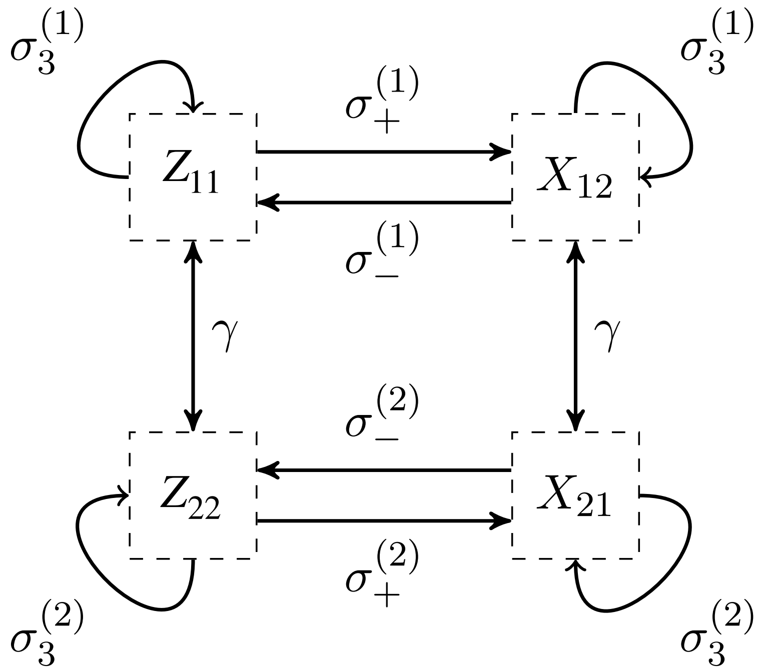
$$\sigma_+^{(2)} Z_{22} = X_{21} \text{ and } \sigma_-^{(1)} X_{21} = Z_{22}.$$

The new generators $\sigma_\pm^{(1)}$ and $\sigma_\pm^{(2)}$ carry the info about the color reps or equivalently the $\mathbb{Z}_2$ symmetry.

The SU(2)$_{XZ}$ sector

For *single letter* words

we are defining an algebroid acting as:

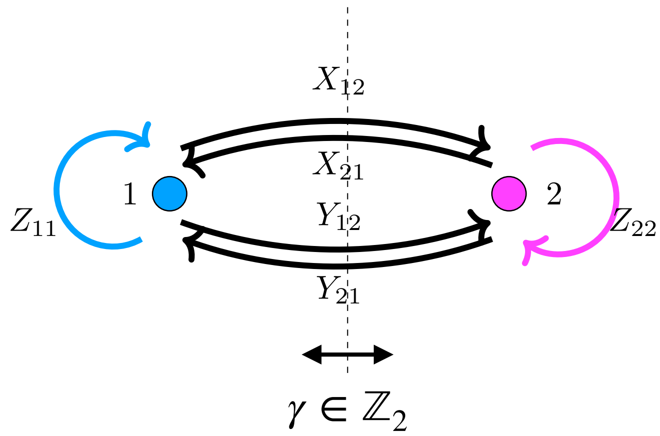


where $\gamma \in \mathbb{Z}_2$ is $\gamma^2 = 1$

$\gamma \triangleright Z_{11} = Z_{22}$ and $\gamma \triangleright X_{12} = X_{21}$

as well as

$\gamma \triangleright Z_{22} = Z_{11}$ and $\gamma \triangleright X_{21} = X_{12}$



The SU(2)$_{XZ}$ sector

For ***two letter*** words it becomes non-trivial:

* <u>$\mathcal{N}=4$ SYM unbroken SU(2):</u> the action on two sites is given by the trivial coproduct $\Delta\sigma_{\pm} = \mathbb{I} \otimes \sigma_{\pm} + \sigma_{\pm} \otimes \mathbb{I}$ which acts as

$$\Delta\sigma_{+} \triangleright XX = XZ + ZX$$

* <u>$\mathcal{N}=2$ SCFT broken SU(2):</u> when using the naive coprdoduct

$$\Delta\sigma_{\pm} = \mathbb{I} \otimes \sigma_{\pm} + \sigma_{\pm} \otimes \mathbb{I}$$

$$\Delta\sigma_{+} \triangleright X_{12}X_{21} = X_{12}Z_{22} + \underline{Z_{11}X_{21}}$$

Not allowed color contraction

The SU(2)$_{XZ}$ sector

✱ For ***two letter*** words it becomes non-trivial:

✱ $\mathcal{N}=4$ SYM unbroken SU(2): the action on two sites is given by the trivial coproduct $\Delta\sigma_\pm = \mathbb{I}\otimes\sigma_\pm + \sigma_\pm\otimes\mathbb{I}$ which acts as

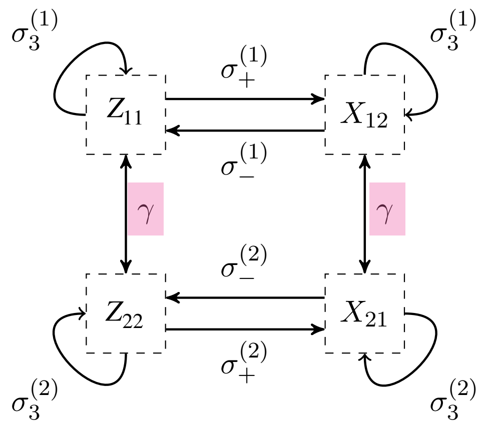
$$\Delta\sigma_+ \triangleright XX = XZ + ZX$$

✱ $\mathcal{N}=2$ SCFT broken SU(2): **improve** the coprdoduct

$\Delta\sigma_\pm = \mathbb{I}\otimes\sigma_\pm + \sigma_\pm\otimes\gamma$ which now gives

$$\Delta\sigma_+ \triangleright X_{12}X_{21} = X_{12}Z_{22} + Z_{11}X_{12}$$

Allowed color contraction



The SU(2)$_{XZ}$ sector

This coproduct naturally generalises for words of any length

$$\Delta\sigma_+ = \sum_{\text{all sites}} \cdots \mathbb{1} \otimes \sigma_+ \otimes \gamma \cdots$$

With this rule, we have a $\mathbb{Z}_2$ SU(2) algebroid obeying the usual SU(2) algebra:

$$\left[\sigma_+^{(n)}, \sigma_-^{(m)}\right] = \sigma_3^{(n)}\delta^{nm} \qquad \left[\sigma_3^{(n)}, \sigma_\pm^{(m)}\right] = \pm\, \sigma_\pm^{(n)}\delta^{nm}$$

Invariance of the Lagrangian

Invariance of the Lagrangian

- Single trace operator like the Lagrangian are traces in color space.

- To act with the coproduct we cut them open using cyclic prescription: $tr(ABC) \to \frac{1}{3}(ABC + BCA + CAB)$

- Explicit calculation the Lagrangian is invariant under the "broken $SU(2)_{XZ}$" as well as the full SU(4). Here we show only SU(2) algebroid.

Invariance of the Lagrangian

The superpotential (at the orbifold point)

$$\mathscr{W} = g\,\mathbf{tr}_1\Big((X_{12}Y_{21} - Y_{12}X_{21})Z_{11}\Big) + g\,\mathbf{tr}_2\Big((X_{21}Y_{12} - Y_{21}X_{12})Z_{22}\Big)$$

after our opening up procedure

$$\frac{1}{g}|\mathscr{W}_1\rangle = \left(X_{12}Y_{21} - Y_{12}X_{21}\right)Z_{11} + Z_{11}\left(X_{12}Y_{21} - Y_{12}X_{21}\right) + \left(Y_{12}Z_{22}X_{21} - X_{12}Z_{22}Y_{21}\right)$$

After acting with any $SU(2)_{XZ}$ generator we get zero

$$\Delta\sigma_+ \triangleright \mathscr{W}_1 \propto \left(X_{12}Y_{21} - Y_{12}X_{21}\right)X_{12} + X_{12}\left(X_{21}Y_{12} - Y_{21}X_{12}\right) + \left(Y_{12}X_{21}X_{12} - X_{12}X_{21}Y_{12}\right) = 0$$

The Kähler part is trivially invariant as it is a singlet under the SU(2).

Away from the orbifold point

✱ The coproduct gets **deformed** by the **marginal deformation** $\kappa = g_2/g_1$.

✱ This deformation is captured by a **Drinfeld-like twist** $\mathcal{F}(\kappa)$.

✱ Which we read of from the F-terms (and D-terms for the full SU(4)).

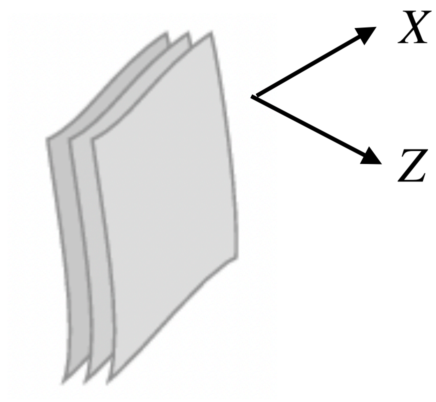
$$\mathcal{F}(\kappa) \triangleright (X_{12}Z_{22} - Z_{11}X_{12}) = X_{12}Z_{22} - \frac{1}{\kappa}Z_{11}X_{12}$$

[2106.08449 Rabe,EP,Zoubos]

Away from the orbifold point

The F-terms define quantum planes $X_{12}Z_{22} - \frac{1}{\kappa}Z_{11}X_{12} = 0$

in the transverse to the D3 branes directions.



[2106.08449 Rabe,EP,Zoubos]

The **B-field** (transverse to the D3) the open strings ending on the D3 branes see a non-commutative geometry. ***A quantum plane***! [Seiberg,Witten1999]

Away from the orbifold point

The twist $\mathscr{F}(\kappa) \triangleright (X_{12}Z_{22} - Z_{11}X_{12}) = X_{12}Z_{22} - \frac{1}{\kappa}Z_{11}X_{12}$

deforms the coproduct $\Delta_\kappa \sigma_\pm = \mathscr{F}(\kappa)\Delta\sigma_\pm \mathscr{F}^{-1}(\kappa)$

$$\Delta_\kappa \sigma_+ = \sum \cdots \mathbb{I} \otimes \sigma_+ \otimes \gamma\kappa^s \cdots$$

Where $s(Z_{11}) = 1 = s(X_{12})$, $s(Z_{22}) = -1 = s(X_{21})$ and $\gamma s = -s\gamma$.

Invariance of the Lagrangian

Away from the orbifold point the superpotential:

$$\mathcal{W} = g_1 \mathrm{tr}_1\Big((X_{12}Y_{21} - Y_{12}X_{21})Z_{11} \Big) + g_2 \mathrm{tr}_2\Big((X_{21}Y_{12} - Y_{21}X_{12})Z_{22} \Big)$$

Opening up

$$\frac{1}{g_1} | \mathcal{W}_1 \rangle = \left(X_{12}Y_{21} - Y_{12}X_{21} \right) Z_{11} + Z_{11} \left(X_{12}Y_{21} - Y_{12}X_{21} \right) + \kappa \left(Y_{12}Z_{22}X_{21} - X_{12}Z_{22}Y_{21} \right)$$

Acting with the marginally deformed coproduct $\Delta\sigma_+ = \sum \cdots \mathbb{I} \otimes \sigma_+ \otimes \gamma\kappa^s \cdots$

$$\Delta\sigma_+ \triangleright \mathcal{W}_1 \propto \left(X_{12}Y_{21} - Y_{12}X_{21} \right) X_{12} + X_{12} \left(X_{12}Y_{21} - Y_{12}X_{21} \right) + \kappa \frac{1}{\kappa} \left(Y_{12}X_{21}X_{12} - X_{12}X_{21}Y_{12} \right) = 0$$

The Kähler part is also invariant as before.

Spectrum

Reps of the SU(2) algebroid

This SU(2) was supposed to be broken. From the point of view of $\mathcal{N}=2$ representation theory, these operators are unrelated!

Using the new coproduct $\mathcal{F}_{sym}(\kappa) = \mathcal{F}_{anti}(1/\kappa)$ they live in the same rep.

$$X_{12}X_{21}X_{12}X_{21}$$

$$\updownarrow$$

$$X_{12}Z_2X_{21}X_{12} + X_{12}X_{21}X_{12}Z_2 + \kappa\,(X_{12}X_{21}Z_1X_{12} + Z_1X_{12}X_{21}X_{12})$$

$$\Delta_\kappa\sigma_+ \quad \updownarrow$$

$$\tfrac{1}{\kappa}X_{12}Z_2Z_2X_{21} + Z_1X_{12}Z_2X_{21} + X_{12}Z_2X_{21}Z_1 + \kappa Z_1Z_1X_{12}X_{21} + \kappa X_{12}X_{21}Z_1Z_1 + \kappa Z_1X_{12}X_{21}Z_1$$

$$\updownarrow \quad \Delta_\kappa\sigma_-$$

$$\tfrac{1}{\kappa}X_{12}Z_2Z_2Z_2 + Z_1X_{12}Z_2Z_2 + \kappa Z_1Z_1X_{12}Z_2 + \kappa^2 Z_1Z_1Z_1X_{12}$$

$$\updownarrow$$

$$\kappa^2 Z_1Z_1Z_1Z_1$$

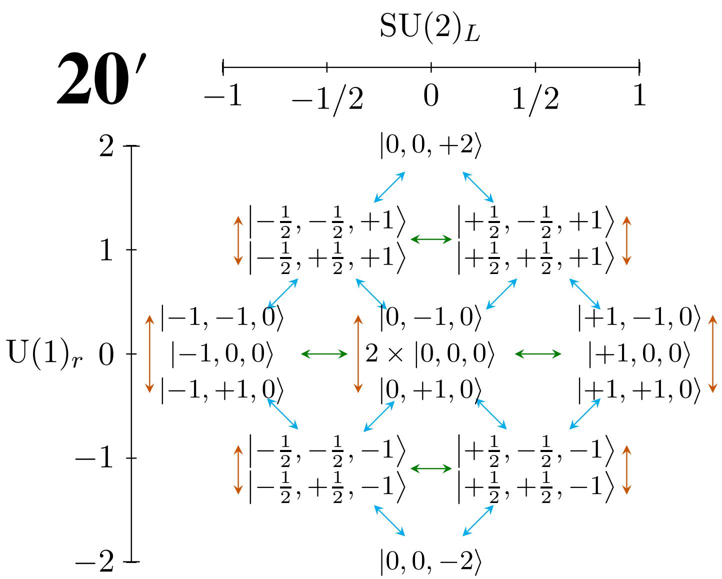
Relating Coulomb and Higgs branch operators in one algebroid multiplet.

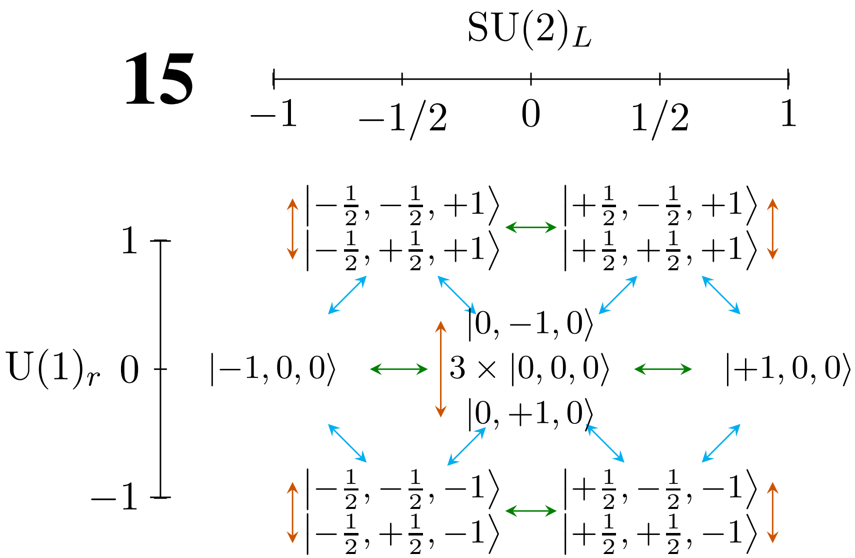
The BPS operators also obtain by diagonalisation of the Hamiltonian (Feynman diagrams).

2-sites for the full SU(4)

$6 \times 6 = \mathbf{20}' + \mathbf{15} + \mathbf{1}$

The $\mathbf{20}'$ is BPS but the $\mathbf{15}$ and the $\mathbf{1}$ are not.





The action of the "broken generators" is denoted by blue arrows.

The unbroken $SU(2)_R$ by orange arrows.

The unbroken $SU(2)_L$ by green arrows.

Summary

✱ Long-range coordinate Bethe ansatz for 3- & 4-magnons eigenvectors.

✱ Infinite tower of Yang-Baxter like equations.

✱ The 4-magnon solution can be written in terms of the 3-magnons one.

✱ Imposing periodic BC on 4-magnons = Hamiltonian diagonalization.

✱ Novel Groupoid symmetry structures: both the Lagrangian & the spectrum.

Future

* Any number of magnons using smearing and poles as a guide.

* Combine with bifundamental vacuum to guess the R-matrix.

* What is the rapidity of the model? [2106.08449 Rabe,EP,Zoubos]

* Better understand & learn how to use the Groupoid symmetries.
[2010.01060 Felder,Ren Quantum Groups for RSOS]

* Relation with RSOS already pointed out in [2106.08449 Rabe,EP,Zoubos].

Thank you!